MATH 6720
Midterm
Due Date: 02/27/2017 1pm

Instructions:

- There is a two hours time limit to complete the midterm. The time limit is enforced on the honor system, do not spend the entire weekend thinking about the problems.
- Verbal or electronic collaborations are not allowed.
- Notes, books, electronic material are not allowed.

1. Show that
$$\int_0^\infty \frac{\cos x - 1}{x^2} dx = -\frac{\pi}{2}.$$
HINT: On one of the contour integral use a Taylor series expansion to obtain an estimate.

2. Consider the complex potential $f(z) = Az^{3/2}$ with $A \in \mathbb{R}$. The resulting flow is called a corner flow (2D irrotational, inviscid, incompressible). Take the principal branch of the logarithm only (positive root) and take the branch cut to be $(0, \infty)$.
   (a) Using the polar representation of z, find the potential Φ and the streamfunction Ψ.
   (b) Find the separatrix of the flow, i.e. the streamlines with $\Psi(r, \theta) = 0$.
   (c) A cylinder is introduced at the origin. Modify the flow so that it flows around the cylinder.
   (d) Calculate the lift and the drag on the cylinder.

3. Find all the possible Laurent expansions of the function $f(z) = \frac{1}{(z-i)(z-2)}$ at $z = 0$.
HINT: First find the different regions where $f(z)$ is analytic.

4. Consider the function $f(z) = \sqrt{z(z-1)}$. Find one branch cut structure and describe $f(z)$ on its branches.

5. Let $f(z)$ be an entire function such that there exists a constant M, an $R > 0$ and an integer $n \geq 1$ with $|f(z)| \leq M|z|^n$ for $|z| > R$. Show that $f(z)$ is a polynomial of degree at most n.