This file contains the problems, suggested for solving on the Russian national mathematical competitions (final part).

I've posted this stuff in a number of articles in rec.puzzles. but I have got many requests for the missing parts. So I have decided to put this material here, having provided it with the answers on the common questions.

==================collected articles start=====================

I'm going to send some problems from the book

Vasil'ev N.B, Egorov A.A. "The problems of the All-Soviet-Union mathematical competitions",-Moscow.:Nauka. 1988 ISBN 5-02-013730-8. (in Russian).

Those problems were submitted for the solving on the competition between the pupils of 8, 9, or 10 forms for 4 hours. So they do not contain something of the advanced topics, -- all of them can be solved by a schoolboy. They can not go out of the common school plan bounds.

Most of the problems are original and the book contains all the necessary references. I am not going to translate all the book, so I shall not send the solutions. Please, accept those messages as they are, to say more exactly -- as I can. I have to do my job, and this is hobby only, but nevertheless, that should be enjoyable to solve those problems.

> "Nobody can embrace the unembraceable."
>
> Kozma Prutkov.
>
> (beginning of the XIX c.)

May be those postings are just a harassment, but I hope, that most of You will not only enjoy the problems solving, but will be able to use them in Your work with the students.

> "Zeal overcomes everything."
>
> "Sometimes zeal overcomes even the common sense."
>
> Kozma Prutkov.

There are some wonderful books in Russian, that have not been translated into English yet, for example,

"Problems of the Moscow mathematical competitions",-compiled by G.A.Galperin, A.K.Tolpygo.,-Moscow, Prosveshchenie, 1986.

A.A.Leman "Collection of Moscow mathematical competitions problems", -Moscow, Prosveshchenie, 1965.

==============answers on the common questions============

WHAT IS THE AGE OF THE PARTICIPANTS?

The Russian pupils start studying being 6-7 years old, so the pupils of the 8th form are about 14.

WHO PARTICIPATE?

The competition helds in 3 - 4 stages
1. At school - if there are many volunteers.
2. Subregional - if the region is big enough.
3. Regional (in some regions, as in Moscow, Leningrad=Sankt-Peterburg, Sverdlovsk=Ekaterinburg or Novosibirsk they are even more interesting and more difficult)
4. Final part, considered in the report.

WHAT ARE THE BEST AND AVERAGE RESULTS?

The winners (2-5) usually give the perfect solution of all the problems with some shortages.

My personal experience refers to that times, when there were two days of the final competition. Than the winners solved all the problems of two days except one problem.

It is very difficult to speak about the average level, because it depends very much on a region, but most of the participants of that time solved at least one problem. The problem is not only the difficulties in the problems themselves, but also in the shortage of time. They successfully solved the problems before the official explanation two days later.

MAY I USE THOSE PROBLEMS IN THE SCHOOL PROJECTS?

You don't need MY permission for using those problems. As concerns the copyright, the usage of all the information in the non-commercial purposes was never restricted in Russia if it is not related to the state security. Moreover, the spirit of the competition encourages everybody to distribute those problems in order to enhance the mathematical culture of the pupils.

And You only are able to decide whether Your students can solve them.

============collected articles continue=================

So... First one -- Moscow, 1961.

```
        form
           8  001   002   003   004   005a
           9  006a  007   008   009   010
          10  011   012   007   006b  005b
```

001.

```
+-------------+--------------+ Given a figure, containing 16 segments.
|             |              | You should prove that there is no curve,
|             |              | that intersect each segment exactly once.
+-------+-----+----+---------+ The curve may be not closed, may
|       |          |         | intersect itself, but it is not
|       |          |         | allowed to touch the segments or
+----------------------------+ to pass through the vertices.
```

002.
Given a rectangle $A_1A_2A_3A_4$. Four circles with A_i as a centres have their radiuses r_1,r_2,r_3,r_4, and $r_1 + r_3 = r_2 + r_4 < d$, where d is a diagonal of the rectangle. Two pairs of the outer common tangents to {the first and the third} and {the second and the fourth} circumferences make a quadrangle.
Prove that You can inscribe a circle into that quadrangle.

003.
Prove that among 39 sequential natural numbers there always is a number with the sum of its digits dividable by 11.

004.
Given a table 4x4.
a) Find, how 7 stars can be put in its fields in such a way, that erasing of two arbitrary lines and two columns will always leave at list one of the stars.
b)Prove that if there are less than 7 stars, You can always find two columns and two rows, that if You erase them, no star will remain in the table.

005.
a)
Given a quartet of positive numbers. (a,b,c,d). It is transformed to the new one according to the rule: $a'=ab$; $b'=bc$; $c'=cd$; $d'=da$.
The second one is transformed to the third according to the same rule and so on.
Prove that if at least one initial number does not

equal 1, than You can never obtain the initial set.

b)

Given a set of 2^k (k-th power of two) numbers, equal either to 1 or to -1. It is transformed as that was in the a) problem (each one is multiplied by the next, and the last -- by the first.

Prove that You will always finally obtain the set of positive units.

006.

a)

Points A and B move uniformly and with equal angle speed along the circumferences with O_a and O_b centres (both clockwise).

Prove that a vertex C of the equilateral triangle ABC also moves along a certain circumference uniformly.

b)

The distance from the point P to the vertices of the equilateral triangle ABC equal |AP|=2, |BP|=3. Find the maximal value of CP.

007.

Given some mxn table, and some numbers in its fields. You are allowed to change the sign in one row or one column simultaneously.

Prove that You can obtain a table, with nonnegative sums over each row and over each column.

008.

Given n points, some of them connected by nonintersecting segments.

You can reach every point from every one, moving through(?) the segments, and there is no couple, connected by two different ways.

Prove that the total number of the segments is (n-1).

009.

Given a,b,p -arbitrary integers.

Prove that there always exist relatively prime (i.e. that have no common divisor) k and l, that (ak + bl) is dividable by p.

010.

Nickolas and Peter are dividing (2n+1) nuts. Each wants to get more.

Three ways for that were suggested. (Each consist of three stages.)

First two stages are common.

```
---------
     1 stage: Peter divides nuts onto 2 heaps,
              each contain not less than 2 nuts.
---------
     2 stage: Nickolas divides both heaps onto 2 heaps,
              each contain not less than 1 nut.
---------
        3 stage:
```

----

1 way: Nickolas takes the biggest and the least heaps.

----

2 way: Nickolas takes two middle size heaps.

----

3 way: Nickolas takes either the biggest and the least heaps or two middle size heaps, but gives one nut to the Peter for the right of choice.

Find the most and the least profitable method for the Nickolas.

011.
Prove that for three arbitrary infinite sequences, of natural numbers

$$a_1, a_2, \dots , a_n, \dots$$
$$b_1, b_2, \dots , b_n, \dots$$
$$c_1, c_2, \dots , c_n, \dots$$

there exist numbers p and q such, that
$a_p \geq a_q$; $b_p \geq b_q$ and $c_p \geq c_q$.

012.
Given 120 unit squares arbitraryly situated in the 20x25 rectangle.
Prove that You can place a circle with the unit
diameter without intersecting any of the squares.

The second competition -- Moscow, 1962.

form					
8	013	014	015	016	017
9	018	019	020	021	017
10	022	023	024	025	026

013.
Given points A',B',C',D', on the continuation of the [AB],[BC],[CD],[DA] sides of the convex quadrangle ABCD, such, that the following pairs of vectors are equal: [BB'[=[AB[, [CC['=[BC[, [DD'=CD[, [AA'[=[DA[.
Prove that the quadrangle A'B'C'D' area is five times more than the quadrangle ABCD area.

014.
Given the circumference s and the straight line l, passing through the centre O of s. Another circumference s' passes through the point O and has its centre on the l. Describe the set of the points M, where the common tangent of s and s' touches s'.

015.

Given positive numbers $a_1, a_2, \ldots, a_{99}, a_{100}$. It is known, that
$a_1 > a_0$; $a_2 = 3a_1 - 2a_0$,
$a_3 = 3a_2 - 2a_1, \ldots, a_{100} = 3a_{99} - 2a_{98}$.
Prove that $a_{100} > 2^{99}$.

016.
Prove that there are no integers a,b,c,d such that the polynomial
$ax^3 + bx^2 + cx + d$ equals 1 at $x=19$, and equals 2 at $x=62$.

017.
Given a $n\times n$ table, where n is odd. There is either 1 or -1 in its every field. A product of the numbers in the column is written under every column. A product of the numbers in the row is written to the right of every row.
Prove that the sum of $2n$ products doesn't equal to 0.

018.
Given two sides of the triangle.
Build that triangle, if medians to those sides are orthogonal.

019.
Given a quartet of positive numbers a,b,c,d, and is known, that $abcd=1$.
Prove that $a^2 + b^2 + c^2 + d^2 + ab + ac + ad + bc + bd + dc \geq 10$

020.
Given right pentagon ABCDE. M is an arbitrary point inside ABCDE or on its side. Let the distances $|MA|, |MB|, \ldots, |ME|$ be renumerated and denoted with $r_1 \leq r_2 \leq r_3 \leq r_4 \leq r_5$.
Find all the positions of the M, giving r_3 the minimal possible value.
Find all the positions of the M, giving r_3 the maximal possible value.

021.
Given 1962 -digit number. It is dividable by 9. Let x be the sum of its digits. Let the sum of the digits of x be y. Let the sum of the digits of y be z. Find z.

022.
The M point is a middle of a isosceles triangle base [AC]. [MH] is orthogonal to [BC] side. Point P is the middle of the segment [MH].
Prove that [AH] is orthogonal to [BP].

023.
What maximal area can have a triangle if its sides a,b,c satisfy inequality $0 \leq a \leq 1 \leq b \leq 2 \leq c \leq 3$?

024.

Given x, y, z, three different integers.
Prove that ($(x-y)^5 + (y-z)^5 + (z-x)^5$) is
dividable by $5(x-y)(y-z)(z-x)$.

025.
Given $a_0, a_1, \dots, a_n$. It is known that $a_0 = a_n = 0$;
$a_{k-1} - 2a_k + a_{k+1} >= 0$ at all $k = 1, 2, \dots, k-1$.
Prove that all the numbers are nonnegative.

026.
Given positive numbers $a_1, a_2, \dots, a_m$; $b_1, b_2, \dots, b_n$;
and is known that $a_1 + a_2 + \dots + a_m = b_1 + b_2 + \dots + b_n$.
Prove that You can fill an empty table with m rows and n columns with
no more than $(m+n-1)$ positive number in such a way, that for all i,j
the sum of the numbers in the i-th row will equal to a_i, and the sum
of the numbers in the j-th column --- to b_j.

The third competition -- Moscow, 1963.

form					
8	027	028	029a	030	031a
9	032	033	034	031b	028
10	035	036	037	029b	028
11	038	028	039	040	029b

027.
Given 5 circumferences, every four of them have a common point.
Prove that there exist a point that belongs to all five circumferences.

028.
Eight men had participated in the chess tournament. (Each meets each;
draws are allowed, giving 1/2 of pont; winner gets 1.) Everyone has
different number of points. The second one has got as many points as
the the four weakest together.
What was the result of the play between the third prizer and
the chess-player that have occupied the seventh place?

029.
a) Each diagonal of the quadrangle halves its area.
Prove that it is a parallelogram.
b) Three main diagonals of the hexagon halve its area.
Prove that they intersect in one point.

030.
Natural numbers a and b are relatively prime. Prove that the greatest

common divisor of $(a+b)$ and (a^2+b^2) is either 1 or 2.

031.
Given two fixed points A and B .The point M runs along the circumference containing A and B. K is the middle of the segment [MB]. [KP] is a perpendicular to the line (MA).
a) Prove that all the possible lines (KP) pass through one point.
b) Find the set of all the possible points P.

032.
Given equilateral triangle with the side 1.
What is the minimal length d of a brush (segment), that will paint all the triangle, if its ends are moving along the sides of the triangle.

033.
A chess-board 6x6 is tiled with the 2x1 dominos.
Prove that You can cut the board onto two parts by a straight line that does not cut dominos.

034.
Given n different positive numbers $a_1, a_2, \ldots, a_n$.
We construct all the possible sums (from 1 to n terms).
Prove that among those sums there are at least $n(n+1)/2$ different ones.

035.
Given a triangle ABC. We build two angle bisectors in the corners A and B. Than we build two lines parallel to those ones through the point C. D and E are intersections of those lines with the bisectors. It happens, that (DE) line is parallel to (AB).
Prove that the triangle is isosceles.

036.
Given the endless arithmetic progression with the positive integer members. One of those is an exact square. Prove that the progression contain the infinite number of the exact squares.

037.
Given right 45-angle. Can You mark its corners with the digits {0,1,...,9} in such a way, that for every pair of digits there would be a side with both ends marked with those digits?

038.
Find such real p,q,a,b, that for all x helds an equality:
$$(2x-1)^{20} - (ax+b)^{20} = (x^2+px+q)^{10}.$$

039.

On the ends of the diameter two "1"s are written. Each of the semicircles is divided onto two parts and the sum of the numbers of its ends (i.e. "2") is written at the middle point. Then every of the four arcs is halved and in its middle the sum of the numbers on its ends is written.
Find the total sum of the numbers on the circumference after n steps.

040.
Given an isosceles triangle. Find the set of the points inside the triangle such, that the distance from that point to the base equals to the geometric mean of the distances to the sides.

The 4-th competition -- Moscow, 1964.

form					
8	041	042	043	044	045a
9	041	046	047	048	049
10	050	051	045ab	052	053
11	054	055	052	053	054

041.
The two heights in the triangle are not less than the respective sides. Find the angles.

042.
Prove that for no natural m a number $m(m + 1)$ is a power of an integer.

043.
Given 1000000000 first natural numbers. We change each number with the sum of its digits an repeat this procedure until there will remain 1000000000 one digit numbers. Is there more 1-s or 2-s?

044.
Given an arbitrary set of $2k+1$ integers $a_1, a_2, \dots, a_{2k+1}$.
We make a new set:

$$\frac{a_1 + a_2}{2}, \frac{a_2 + a_3}{2}, \dots, \frac{a_{2k} + a_{2k+1}}{2}, \frac{a_{2k+1} + a_1}{2},$$

and a new one, according to the same rule, and so on...
Prove that if we obtain integers only, the initial set consisted of equal integers only.

045.
a) Given a convex hexagon ABCDEF with all the equal angles.
Prove that $|AB|-|DE| = |EF|-|BC| = |CD|-|FA|$.

b) The opposite problem: Prove that it is possible to build a convex hexagon with equal angles of six segments $a_1, a_2, \dots, a_6$, whose lengths satisfy the condition $a_1 - a_4 = a_5 - a_2 = a_3 - a_6$.

046.
Find integer solutions (x,y) of the equation (1964 times "sqrt"):
$\sqrt{x + \sqrt{x + \sqrt{\dots(x + \sqrt{x})\dots}}} = y$.

047.
Four perpendiculars are drawn from the vertices of a convex quadrangle to its diagonals.
Prove that their bases make a quadrangle similar to the given one.

048.
Find all the natural n such that n! is not dividable by n^2.

049.
A honeybug crawls along the honeycombs with the unite length of their hexagons. He has moved from the node A to the node B along the shortest possible trajectory.
Prove that the half of his way he moved in one direction.

050.
The quadrangle ABCD is outscribed around the circle with the centre O. Prove that the sum of AOB and COD angles equals 180 degrees.

051.
Given natural a,b,n. It is known, that for every natural k ($k \neq b$) the number $(a - k^n)$ is dividable by $(b-k)$. Prove that $a = b^n$.

052.
Given an expression $x_1 : x_2 : \dots : x_n$ (: means division).
We can put the braces as we want. How many expressions can we obtain?

053.
We have to divide a cube onto k nonoverlapping tetrahedrons.
For what smallest k is it possible?

054.
Find the smallest exact square with last digit not 0, such that after deleting its last two digits we shall obtain another exact square.

055.
Let ABCD be an outscribed trapezoid; E is a point of its diagonals intersection; r_1, r_2, r_3, r_4 -- the radiuses of the circles inscribed in the triangles ABE, BCE, CDE, DAE respectively.

Prove that $\frac{1}{r_1} + \frac{1}{r_3} = \frac{1}{r_2} + \frac{1}{r_4}$.

The fifth competition -- Moscow, 1965.

form					
8	056a	057	058	059	060
9	061	062	063	064	065
10	056b	066	067a	068a	069
11	063	067b	070	068b	071

056.

a) Each of the numbers $x_1, x_2, \dots, x_n$ can be 1, 0, or -1. What is the minimal possible value of the sum of all products of couples of those numbers.

b) Each absolute value of the numbers $x_1, x_2, \dots, x_n$ is less than 1. What is the minimal possible value of the sum of all products of couples of those numbers.

057.

Given a board 3x3 and 9 cards with some numbers (known to the players). Two players, in turn, put those cards on the board. The first wins if the sum of the numbers in the first and the third row is greater than in the first and the third column. Prove that it doesn't matter what numbers are on the cards, but if the first plays the best way, the second can not win.

058.

A circle is outscribed around the triangle ABC. Chords, from the middle of the arc AC to the middles of the arcs AB and BC, intersect sides [AB] and [BC] in the points D and E. Prove that (DE) is parallel to (AC) and passes through the centre of the inscribed circle.

059.

A bus ticket is considered to be lucky if the sum of the first three digits equals to the sum of the last three (6 digits in Russian buses). Prove that the sum of all the lucky numbers is dividable by 13.

060.

There is a lighthouse on a small island. Its lamp enlights a segment of a sea to the distance a. The light is turning uniformly, and the end of the segment moves with the speed v.
Prove that a ship, whose speed doesn't exceed v/8 cannot arrive to the

island without being enlightened.

061.
A society created in the help to the police contains 100 man exactly.
Every evening 3 men are on duty. Prove that You can not organise
duties in such a way, that every couple will meet on duty once exactly.

062.
What is the maximal possible length of the segment, being cut out by
the sides of the triangle on the tangent to the inscribed circle, being
drawn parallel to the base, if the triangle's perimeter equals 2p?

063.
Given n^2 numbers x_{i,j} (i,j = 1,2,...,n) satisfying the system of
n^3 equations: x_{i,j} + x_{j,k} + x{k,i} = 0 (i,j,k = 1,...,n).
Prove that there exist such a numbers a_1, a_2, ... , a_n;
that for all i,j = 1,...n   x{i,j}= a_i - a_j.

064.
Is it possible to pose 1965 points inside the unit square to make every
rectangle with the square 0.005 and sides parallel to the side of the
square to contain at least one of those points?

065.
Quasi-rounding is a substitution one of the two closest integers
instead of the given number.
Given n numbers. Prove that You can quasi-round them in such a way,
that a sum of every subset of quasi-rounded numbers will deviate from
the sum of the same subset of initial numbers not greater than (n+1)/4.

066.
The tourist has come to the Moscow by train. All-day-long he wandered
randomly through the streets. Than he had a supper in the cafe on the
square and decided to return to the station only through the streets
that he has passed an odd number of times.
Prove that he is always able to do that.

067.
a) A certain committee has gathered 40 times. There were 10 members on
   every meeting. Not a single couple has met on the meetings twice.
   Prove that there were no less then 60 members in the committee.
b) Prove that You can not construct more then 30 subcommittees
   of 5 members from the committee of 25 members, with no
   couple of subcommittees having more than one common member.

068.

Given two relatively prime numbers $p>0$ and $q>0$.
An integer n is called "good" if we can represent it as $n = px + qy$ with nonnegative integers x and y, and "bad" in the opposite case.

a) Prove that there exist integer c such that in a pair $\{n, c-n\}$ always one is "good" and one is "bad".
b) How many there exist "bad" numbers.?

069.
An airplane-spy flies on the circumference with the centre A and radius 10 km. Its speed is 1000 km/h. At the certain moment a rocket with the same speed starts from the point A and moves remaining on one line with the plain and A. What time is necessary for it to hit the plane?
/* remember, no advanced math. were known to the participants -VAP */

070.
Prove that the sum of the lengths of the polyhedron edges exceeds its tripled diameter (distance between two farest vertices).

071.
On the surface of the planet lives one inhabitant, that can move with the speed not greater than u. A spaceship approaches to the planet wit its speed v. Prove that if $v/u > 10$, the spaceship can find the inhabitant, even it is trying to hide.

The sixth competition -- Voronezh, 1966.

form					
8	072	073a	074	075a	076
9	077	073b	075b	078	079
10,11	075b	080	081	082	083

072.
There is exactly one astronomer on every planet of a certain system. He watches the closest planet. The number of the planets is odd and all of the distances are different.
Prove that there one planet being not watched.

073.
a) Points B and C are inside the segment [AD]. $|AB|=|CD|$.
   Prove that for all of the points P on the plane holds unequality $|PA|+|PD| > |PB|+|PC|$.

b) Given four points A,B,C,D on the plane. For all of the points P on the plane holds unequality $|PA|+|PD| > |PB|+|PC|$.
   Prove that points B and C are inside the segment [AD] and $|AB|=|CD|$.

074.
Can both (x^2+y) and (y^2+x) be exact squares for natural x and y?

075.
a) Pupils of the 8-th form are standing in a row. There is the pupil of the 7-th form in before each, and he is smaller (in height) than the elder. Prove that if You will sort the pupils in each of rows with respect to their height, every 8-former will still be taller than the 7-former standing before him.
b) An infantry detachment soldiers stand in the rectangle, being arranged in columns with respect to their height. Prove that if You rearrange them with respect to their height in every separate row, they will still be staying with respect to their height in columns.

076.
A rectangle ABCD is drawn on the cross-lined paper with its sides laying on the lines, and $|AD|$ is k times more than $|AB|$ (k is an integer). All the shortest paths from A to C coming along the lines are considered. Prove that the number of those with the first link on [AD] is k times more then of those with the first link on [AB].

077.
Given the numbers $a_1, a_2, \dots , a_n$, such that
$0 \le a_1 \le a_2 \le 2a_1$, $a_2 \le a_3 \le 2a_2$, ... ,
$a_{n-1} \le a_n \le 2a_{n-1}$,
Prove that in the sum $s = \pm a_1 \pm a_2 \pm \dots \pm a_n$
You can choose appropriate signs to make $0 \le s \le a_1$.

078.
Prove that You can always pose a circle of radius S/P inside a convex polygon with the perimeter P and area S.

079.
For three arbitrary crossroads A,B,C in a certain city there exist a way from A to B not coming through C. Prove that for every couple of the crossroads there exist at least two non-intersecting ways connecting them. (there are at least two crossroads in the city)

080.
Given a triangle ABC. Consider all the tetrahedrons PABC with PH -- the smallest of all tetrahedron's heights. Describe the set of all possible points H.

081.
Given 100 points on the plain. Prove that You can cover them with a

family of circles with the sum of their diameters less than 100 and the distance between any two of the circles more than one.

082.
The distance from A to B is d kilometres. A plain flying with the constant speed in the constant direction along and over the line (AB) is being watched from those points. Observers have reported that the angle to the plain from the point A has changed by \alpha degrees and from B --- by \betha degrees within one second.
What can be the minimal speed of the plain?

083.
20 Numbers are written on the board: 1, 2, ... ,20. Two players are putting signs before the numbers in turn (+ or -). The first wants to obtain the minimal possible absolute value of the sum.
What is the maximal value of the absolute value of the sum that can be achieved by the second player?

"Where is the beginning of that end,
that ends the beginning?"
Kozma Prutkov.

"If You see a title 'buffalo'
on the elephant's cage
-- don't believe to Your eyes!"
Kozma Prutkov.

The name has been changed -- the numeration was restarted.

The first competition -- Tbilisi, 1967.

form					
8	084a	085a	086a	087	088
9	087b	086a	085b	084b	088
10	090	086b	091	092	093

084.
a) The maximal height |AH| of the acute-angled triangle ABC equals the median |BM|. Prove that the angle ABC isn't greater than 60 degrees.
b) The height |AH| of the acute-angled triangle ABC equals the median |BM| and bisectrix |CD|. Prove that the angle ABC is equilateral.

085.
a) The digits of a natural number were rearranged. Prove that the sum of given and obtained numbers can't equal 999...9 (1967 of nines).
b) The digits of a natural number were rearranged. Prove that if the

sum of the given and obtained numbers equals $\{10\}^{10}$, than the given number was dividable by 10.

086.

a) A lamp of a lighthouse enlights an angle of 90 degrees. Prove that You can turn the lamps of four arbitrary posed lighthouses so, that all the plane will enlighted.

b) There are eight lamps in eight points of the space. Each can enlight an octant (three-faced space angle with three mutually orthogonal edges).

Prove that You can turn them so, that all the space will be enlighted.

087.

a) Can You pose the numbers 0,1,...,9 on the circumference in such a way, that the difference between every two neighbours would be either 3 or 4 or 5?

b) The same question, but about the numbers 0,1,...,13.

088.

Prove that there exists a number dividable by 5^{1000} not containing a single zero in its decimal notation.

089.

Find all the integers x,y satisfying equation
$x^2 + x = y^4 + y^3 + y^2 + y$.

090.

In the sequence of the natural (i.e. positive integers) numbers every member from the third equals the absolute value of the difference of the two previous. What is the maximal possible length of such a sequence, if every member is less or equal to 1967?

091.

"KING-THE SUICIDER"

Given a chess-board 1000x1000, 499 white castles and a black king. Prove that it does not matter neither the initial situation nor the way white plays, but the king can always move and stay under the check.

092.

Three vertices KLM of the rhombus (diamond) KLMN lays on the sides [AB], [BC] and [CD] of the given unit square. Find the area of the set of all the possible vertices N.

093.

Given natural number k with a property "if n is dividable by k, than the number, obtained from n by reversing the order of its digits is also dividable by k". Prove that the k is a divisor of 99.

The second competition -- Leningrad, 1968.

form	first day					second day				
8	094	095	096	097	098	105a	106	107	108	109
9	099	100	101	097	102	110	111	105a	108	109
10	103	095	104	097	096	105b	112	113	114	109

094.
Given an octagon with the equal angles. The lengths of all the sides are integers. Prove that the opposite sides are equal in pairs.

095.
What is greater, $\{31\}^{11}$ or $\{17\}^{14}$?
/* calculators were not available - VAP */

096.
The circumference with the radius 100cm is drawn on the cross-lined paper with the side of the squares 1cm. It neither comes through the vertices of the squares, nor touches the lines. How many squares can it pass through?

097.
Some students on the faculty speak several languages and some - Russian only. 50 of them know English, 50 -- French and 50 -- Spanish. Prove that it is possible to divide them onto 5 groups, not necessary equal, to get 10 of them knowing English, 10 -- French and 10 -- Spanish in each group.

098.
Prove the equality

$$\frac{2}{x^2 - 1} + \frac{4}{x^2 - 4} + \frac{6}{x^2 - 9} + \dots + \frac{20}{x^2 - 100} =$$

$$= 11 \left(\frac{1}{(x-1)(x+10)} + \frac{1}{(x-2)(x+9)} + \dots + \frac{1}{(x-10)(x+1)} \right).$$

099.
The difference between the maximal and the minimal diagonals of the right n-angle equals to its side (n > 5). Find n.

100.

The sequence $a_1, a_2, a_3, \ldots$, is built according to the rule
$a_1 = 1$, $a_2 = a_1 + 1/a_1$, ... , $a_{n+1} = a_n + 1/a_n$, ...
Prove that $a_{100} > 14$.

101.
Given two acute-angled triangles ABC and $A'B'C'$ with the points O and O' inside. Three pairs of the perpendiculars are drawn:
$[OA_1]$ to the side $[BC]$, $[O'A'_1]$ to the side $[B'C']$,
$[OB_1]$ to the side $[AC]$, $[O'B'_1]$ to the side $[A'C']$,
$[OC_1]$ to the side $[AB]$, $[O'C'_1]$ to the side $[A'B']$;
it is known that
$[OA_1]$ is parallel to the $[O'A']$,
$[OB_1]$ is parallel to the $[O'B']$,
$[OC_1]$ is parallel to the $[O'C']$ and
$|OA_1| \cdot |O'A'| = |OB_1| \cdot |O'B'| = |OC_1| \cdot |O'C'|$.
Prove that
$[O'A'_1]$ is parallel to the $[OA]$,
$[O'B'_1]$ is parallel to the $[OB]$,
$[O'C'_1]$ is parallel to the $[OC]$ and
$|O'A'_1| \cdot |OA| = |O'B'_1| \cdot |OB| = |O'C'_1| \cdot |OC|$.

102.
Prove that You can represent an arbitrary number not exceeding $n!$ (n-factorial; $n! = 1 \cdot 2 \cdot 3 \cdot \ldots \cdot n$) as a sum of k different numbers ($k \le n$) that are divisors of $n!$.

103.
Given a triangle ABC, point D on $[AB]$, E on $[AC]$; $|AD| = |DE| = |AC|$, $|BD| = |AE|$, DE is parallel to BC. Prove that the length $|BD|$ equals to the side of a right decagon (ten-angle) inscribed in a circle with the radius $R=|AC|$.

104.
Three spheres are built so that the edges $[AB]$, $[BC]$, $[AD]$ of the tetrahedron $ABCD$ are their respective diameters.
Prove that the spheres cover all the tetrahedron.

105.
a)

+	-	+	+
+	+	+	+
+	+	+	+

The fields of the square table 4x4 are filled with the "+" or "-" signs. You are allowed to change the signs simultaneously in the whole row, column, or diagonal to the opposite sign. That means, for example, that You can change the sign in the corner square, because it makes a diagonal itself. Prove that You

```
|  +  |  +  |  +  |  +  | will never manage to obtain a table
+-----+-----+-----+-----+ containing pluses only.
```

b)

The fields of the square table 8x8 are filled with the "+" or signs except one non-corner field with "-". You are allowed to change the signs simultaneously in the whole row, column, or diagonal to the opposite sign. That means, for example, that You can change the sign in the corner field, because it makes a diagonal itself. Prove that You will never manage to obtain a table containing pluses only.

106.

Medians divide the triangle onto 6 smaller ones. 4 of the circles inscribed in those small ones are equal.
Prove that the triangle is equilateral.

107.

Prove that the equation $x^2 + x + 1 = py$ has solution (x,y) for the infinite number of simple p.

108.

Each of the 9 referees on the figure skating championship estimates the program of 20 sportsmen by assigning him a place (from 1 to 20). The winner is determined by adding those numbers. (The less is the sum - the higher is the final place).
It was found, that for the each sportsman, the difference of the places, received from the different referees was not greater than 3. What can be the maximal sum for the winner?

109.

Two finite sequences $a_1, a_2, \dots , a_n$; $b_1, b_2, \dots , b_n$ are just rearranged sequence $1, 1/2, \dots , 1/n$.
$a_1 + b_1 \geq a_2 + b_2 \geq \dots \geq a_n + b_n$.
Prove that for every m ($1 \leq m \leq n$) $a_m + a_n \geq 4/m$.

110.

There is scales on the teacher's table. There is a set of weighs on the scales, and there are some pupils' names (may be more than one) on the every weigh. A pupil entering the classroom moves all the weight with his name to another side of the scales. Prove that You can let in such a subset of the pupils, that the scales will change its position.

111.

The city is a rectangle divided onto squares by m streets coming from the West to the East and n streets coming from the North to the South. There are militioners (policemen) on the streets but not on the crossroads. They watch the certain automobile, moving along the closed

route, marking the time and the direction of its movement. Its trace is not known in advance, but they know, that it will not pass over the same segment of the way twice.
What is the minimal number of the militioners providing the unique determination of the route according to their reports?

112.
The circle inscribed in the triangle ABC touches the side [AC] in the point K. Prove that the line connecting the middle of the [AC] side with the centre of the circle halves the [BK] segment.

113.
The sequence $a_1, a_2, \dots, a_n$ satisfies the following conditions:
$a_1 = 0$, $|a_2| = |a_1 + 1|$, ... , $|a_n| = |a_{n-1} + 1|$.

Prove that $\dfrac{a_1 + a_2 + \dots + a_n}{n} \geq -\dfrac{1}{2}$.

114.
Given a quadrangle ABCD. The lengths of all its sides and diagonals are the rational numbers. Let O be the point of its diagonals intersection. Prove that $|AO|$ - the length of the [AO] segment is also rational.

The third competition -- Kiev, 1969.

form	first day			second day		
8	115	116	117	122	123	124a
9	118	119	115	124	125	126
10	119	120	121	125	126	128

115.
The point E lies on the base [AD] of the trapezoid ABCD. The triangles ABE, BCE and CDE perimeters are equal. Prove that $|BC| = |AD|/2$.

116.
There is a wolf in the centre of a square field, and four dogs in the corners. The wolf can easyly kill one dog, but two dogs can kill the wolf. The wolf can run all over the field, and the dogs -- along the fence (border) only. Prove that if the dog's speed is 1.5 times more than the wolf's, than the dogs can prevent the wolf escaping.

117.
Given a finite sequence of 0's and 1's with two properties:
-- if you chose five sequential digits in one place and in the second place, those will be two different binary numbers.

(Some last digits of the first number may be included as
the first digits in the second.)

-- if You add 0 or 1 either from the left or from the right side, the previous property will not be held.

Prove that the first four digits of that sequence coincide with the last four.

118.

Given positive numbers a,b,c,d.
Prove that the set of unequalities

$a + b < c + d$;

$(a + b)(c + d) < ab + cd$;

$(a + b)cd < ab(c + d)$

contain at least one wrong.

119.

For what minimal natural a the polynomial $ax^2 + bx + c$ with the integer c and b has two different positive roots both less than one.

120.

Given natural n. Consider all the fractions $1/(pq)$, where p and q are relatively prime; $0 < p < q \le n$; $p+q > n$.
Prove that the sum of all such a fractions equals to 1/2.

121.

Given n points in the three dimensional space such, that the arbitrary triangle with the vertices in three of those points contains an angle greater than 120 degrees.
Prove that You can rearrange them to make a polyline (unclosed) with all the angles between the sequent links greater than 120 degrees.

122.

Find four different three-digit decimal numbers starting with the same digit, such that their sum is dividable by three of them.

123.

Every city in the certain state is connected by airlines with no more than with three other ones, but one can get from every city to every other city changing a plane once only or directly.
What is the maximal possible number of the cities?

124.

Given a pentagon with all equal sides.

a) Prove that there exist such a point on the maximal diagonal, that every side is seen from it inside a right angle.
/* I mean that the side AB is seen from the point C inside an

arbitrary angle that is greater or equal than angle ACB. - VAP */
b) Prove that the circles build on its sides as on the diameters cannot cover the pentagon entirely.

125.
Given an equation $x^3 + ?x^2 + ?x + ? = 0$. First player substitutes an integer on the place of one of the interrogative marks, than the same do the second with one of the two remained marks, and, finally, the first puts the integer instead of the last mark.
Explain how can the first provide the existence of three integer roots in the obtained equation. (The roots may coincide.)

126.
20 football teams participate in the championship. What minimal number of the games should be played to provide the property: from the three arbitrary teams we can find at least on pair that have already met in the championship.

127.
Let h_k be an apothem of the right k-angle inscribed into a circle with radius R. Prove that $(n + 1)h_{n+1} - nh_n > R$.

128.
Prove that for the arbitrary positive $a_1, a_2, \dots, a_n$ helds unequality

$$\frac{a_1}{a_2 + a_3} + \frac{a_2}{a_3 + a_4} + \dots + \frac{a_{n-1}}{a_n + a_1} + \frac{a_n}{a_1 + a_2} > \frac{n}{4}.$$

....

The 4-th competition -- Simferopol, 1970.

form	first day					second day		
8	129	130	131	132	133a			
9	134	135	133b	136	137			
10	138	139	133b	136	140	141	142	143

129.
Given a circle, its diameter [AB] and a point C on it. Build (with the help of compasses and ruler) two points X and Y, that are symmetric with respect to (AB) line, such that (YC) is orthogonal to (XA).

130.
The product of three positive numbers equals to one, their sum is strictly greater than the sum of the inversed numbers.

Prove that one and only one of them is greater than one.

131.
How many sides of the convex polygon can equal its longest diagonal?

132.
The digits of the 17-digit number are rearranged in the reverse order.
Prove that at list one digit of the sum of the new and the initial number is even.

133.
a) A castle is equilateral triangle with the side of 100 metres. It is divided onto 100 triangle rooms. Each wall between the rooms is 10 metres long and contain one door. You are inside and are allowed to pass through every door not more than once.
Prove that You can visit not more than 91 room (not exiting the castle).
b) Every side of the triangle is divided onto k parts by the lines parallel to the sides. And the triangle is divided onto k^2 small triangles. Let us call the "chain" such a sequence of triangles, that every triangle in it is included only once, and the consecutive triangles have the common side.
What is the greatest possible number of the triangles in the chain?

134.
Given five segments. It is possible to build a triangle of every subset of three of them.
Prove that at least one of those triangles is acute-angled.

135.
The bisector [AD], the median [BM] and the height [CH] of the acute-angled triangle ABC intersect in one point.
Prove that the angle BAC is greater than 45 degrees.

136.
Given five n-digit binary numbers. For each two numbers their digits coincide exactly on m places. There is no place with the common digit for all the five numbers.
Prove that $\frac{2}{5} \le \frac{m}{n} \le \frac{3}{5}$.

137.
Prove that from every set of 200 integers You can choose a subset of 100 with the total sum dividable by 100.

138.

Given triangle ABC, middle M of the side [BC], the centre O of the inscribed circle. The line (MO) crosses the height AH in the point E. Prove that the distance $|AE|$ equals the inscribed circle radius.

139.
Prove that for every natural number k there exists an infinite set of such natural numbers t, that the decimal notation of t does not contain zeroes and the sums of the digits of the numbers t and kt are equal.

140.
Two equal rectangles are intersecting in 8 points. Prove that the common part area is greater than the half of the rectangle’s area.

141.
All the 5-digit numbers from 11111 to 99999 are written on the cards. Those cards lies in a line in an arbitrary order.
Prove that the resulting 444445-digit number is not a power of two.

142.
All natural numbers containing not more than n digits are divided onto tcompwo groups. The first contains the numbers with the even sum of the digits, the second -- with the odd sum. Prove that if $0 < k < n$ than the sum of the k-th powers of the numbers in the first group equals to the sum of the k-th powers of the numbers in the second group.

143.
The vertices of the right n-angle are marked with some colours (each vertex -- with one colour) in such a way, that the vertices of one colour represent the right polygon.
Prove that there are two equal ones among the smaller polygons.

The 5-th competition -- Riga, 1971.

form	first day					second day		
8	144	145a	146a	147		152ab	153	154
9	144	145a	148	147	146b	156abc	152c	155
10	149	145b	150	147	151b	156	157	158

144.
Prove that for every natural n there exists a number, containing only digits "1" and "2" in its decimal notation, that is dividable by 2^n (n-th power of two).

145.
a) Given a triangle $A_1A_2A_3$ and the points B_1 and D_2 on the side

$[A_1A_2]$, B_2 and D_3 on the side $[A_2A_3]$, B_3 and D_1 on the side $[A_3A_1]$. If You build parallelograms $A_1B_1C_1D_1$, $A_2B_2C_2D_2$ and $A_3B_3C_3D_3$, the lines (A_1C_1), (A_2C_2) and (A_3C_3), will cross in one point O. Prove that if $|A_1B_1| = |A_2D_2|$ and $|A_2B_2| = |A_3D_3|$, than $|A_3B_3| = |A_1D_1|$.

b) Given a convex polygon $A_1A_2 \dots A_n$ and the points
B_1 and D_2 on the side $[A_1A_2]$,
B_2 and D_3 on the side $[A_2A_3]$,
....
B_n and D_1 on the side $[A_nA_1]$.
If You build parallelograms $A_1B_1C_1D_1$, $A_2B_2C_2D_2$ ... , $A_nB_nC_nD_n$, the lines (A_1C_1), (A_2C_2), ..., (A_nC_n), will cross in one point O. Prove that
$|A_1B_1|*|A_2B_2|*\dots*|A_nB_n| = |A_1D_1|*|A_2D_2|*\dots*|A_nD_n|$.

146.

a) A game for two. The first player writes two rows of ten numbers each, the second under the first. He should provide the following property: if number b is written under a, and d -- under c, then $a + d = b + c$. The second player has to determine all the numbers. He is allowed to ask the questions like "What number is written in the x place in the y row?" What is the minimal number of the questions asked by the second player before he founds out all the numbers?

b) There was a table mxn on the blackboard with the property: if You chose two rows and two columns, then the sum of the numbers in the two opposite vertices of the rectangles formed by those lines equals the sum of the numbers in two another vertices. Some of the numbers are cleaned. but it is still possible to restore all the table. What is the minimal possible number of the remaining numbers?

147.

Given an unite square and some circles inside. Radius of each circle is less than 0.001, and there is no couple of points belonging to the different circles with the distance between them 0.001 exactly.
Prove that the area, covered by the circles is not greater than 0.34.

148.

The volumes of the water containing in each of three big enough containers are integers. You are allowed only to relocate some times from one container to another the same volume of the water, that the destination already contains.
Prove that You are able to discharge one of the containers.

149.

Prove that if the numbers p_1, p_2, q_1, q_2 satisfy the condition
$(q_1 - q_2)^2 + (p_1 - p_2)(p_1q_2 - p_2q_1) < 0$, then the square

polynomials $x^2 + p_1x + q_1$ and $x^2 + p_2x + q_2$ have real roots, and between the roots of each there is a root of another one.

150.
The projections of the body on two planes are circles.
Prove that they have the same radius.

151.
Some numbers are written along the ring. If unequality $(a-d)(b-c) < 0$ helds for the four arbitrary numbers in sequence a,b,c,d, You have to change the numbers b and c places.
Prove that You will have to do this operation finite number of times.

152.
a) Prove that the line dividing the triangle onto two polygons with equal perimeters and equal areas passes through the centre of the inscribed circle.
b) Prove the same statement for the arbitrary polygon outscribed around the circle.
c) Prove that all the lines halving its perimeter and area simultaneously, intersect in one point.

153.
Given 25 different positive numbers. Prove that You can choose two of them such, that none of the other numbers equals neither to the sum nor to the difference between the chosen numbers.

154.
a) The vertex A_1 of the right 12-angle (dodecagon) $A_1A_2...A_{12}$ is marked with "-" and all the rest -- with "+". You are allowed to change the sign simultaneously in the 6 vertices in succession. Prove that is impossible to obtain dodecagon with A_2 marked with "-" and the rest of the vertices -- with "+".
b) Prove the same statement if it is allowed to change the signs not in six, but in four vertices in succession.
c) Prove the same statement if it is allowed to change the signs in three vertices in succession.

155.
N unit squares on the infinite sheet of cross-lined paper are painted with black colour. Prove that You can cut out the finite number of square pieces and satisfy two conditions
- all the black squares are contained in those pieces.
- the area of black squares is not less than 1/5 and not greater than 4/5 of every piece area.

The task for the tenth form was as follows:
"You are given three serious problems. Try to investigate at least one, but to obtain as many results, as You can. At the end of Your work make a sort of resume, showing the main proved facts, challenged examples and the hypotheses that seem to be true ..."

/* this form of competition was never repeated later - it had required too much efforts from those who checked the works */

156.
A cube with the edge of length n is divided onto n^3 unit ones. Let us choose some of them and draw three lines parallel to the edges through their centres. What is the least possible number of the chosen small cubes necessary to make those lines cross all the smaller cubes?
a) Find the answer for the small n ($n = 2,3,4$).
b) Try to find the answer for $n = 10$.
c) If You can not solve the general problem, try to estimate that value from the upper and lower side.
d) Note, that You can reformulate the problem in such a way: Consider all the triples (x_1,x_2,x_3), where x_i can be one of the integers $1,2,...,n$. What is the minimal number of the triples necessary to provide the property: for each of the triples there exist the chosen one, that differs only in one coordinate. Try to find the answer for the situation with more than three coordinates, for example, with four.

157.
a) Consider the function $f(x,y) = x^2 + xy + y^2$. Prove that for the every point (x,y) there exist such integers (m,n), that $f((x-m),(y-n)) \le 1/2$.
b) Let us denote with $g(x,y)$ the least possible value of the $f((x-m),(y-n))$ for all the integers m,n. The statement a) was equal to the fact $g(x,y) \le 1/2$. Prove that in fact, $g(x,y) \le 1/3$. Find all the points (x,y), where $g(x,y)=1/3$.
c) Consider function $f_a(x,y) = x^2 + axy + y^2$ ($0 \le a \le 2$). Find any c such that $g_a(x,y) \le c$. Try to obtain the closest estimation.

158.
```
  1   2         1   2
  |   |         |   |      The switch with two inputs and two
  v   v         v   v      outputs can be in one of two
+-+---+-+    +-+---+-+   different positions. In the left
| |   | |    | |   | |   part of the picture a) the first input
|  \ /  |    | |   | |   is connected with the second output
```

```
|   X   |    | |   | |   and we can denote this as
|  / \  |    | |   | |   1 2
| |   | |    | |   | |   | |   and the position in the right
+-+---+-+    +-+---+-+   V V   part of the picture   1 2
  |   |        |   |     2 1   will be denoted as    | |
  V   V        V   V                                 V V
  1   2        1   2                                 1 2
     fig.a)
```

```
  1   2   3           The scheme on the picture b) is universal
  |   |   |           in that sense that changing the state of
  V   V   |           the element switches You can obtain all
+-+---+-+ |           the six connections, i.e.
|       | |
|       | |           1 2 3   1 2 3   1 2 3   1 2 3   1 2 3   1 2 3
+-+---+-+ |           | | |   | | |   | | |   | | |   | | |   | | |
  |   |   |           V V V   V V V   V V V   V V V   V V V   V V V
  |   V   V           1 2 3   1 3 2   2 1 3   2 3 1   3 1 2   3 2 1
  | +-+---+-+
  | |       |         (Check it. Note, that the total number of the
  | |       |         states is 2^3 = 8, because each element
  | +-+---+-+         can be in two positions.)
  |   |   |
  V   V   |
+-+---+-+ |
|       | |
|       | |
+-+---+-+ |
  |   |   |
  V   V   V
  1   2   3
     fig.b)
```

a) Try to build the universal scheme for 4 inputs and 4 outputs,
   that can provide all of 24 possible connections.

b) What is the minimal number of the element switches for such a
   scheme?

Let us call a scheme with n inputs and n outputs n-universal, if it can provide all n! possible connections of n inputs with n outputs.

c) Here is the scheme (picture c) with 8 inputs and 8 outputs, where A
   and B are 4-universal. Prove that it is 8-universal.

```
  1        2        3        4            5        6        7        8
  |        |        |        |            |        |        |        |
  V  V     V  V     V  V     V  V      V  V     V  V     V  V     V  V
+-+--+-+ +-+--+-+ +-+--+-+ +-+--+-+  +-+--+-+ +-+--+-+ +-+--+-+ +-+--+-+
```

```
|      | |      | |      | |      |   |      | |      | |      | |      |
|      | |      | |      | |      |   |      | |      | |      | |      |
+-+--+-+ +-+--+-+ +-+--+-+ +-+--+-+   +-+--+-+ +-+--+-+ +-+--+-+ +-+--+-+
  |    \   |    \   |    \   |    \   /    |   /    |   /    |   /    |
  |     \  |     \  |     \  |     \ /     |  /     |  /     |  /     |
  |      \ |      \ |      \ |      X      | /      | /      | /      |
 ......................................................................

  |/      |/      |/      |/           \|      \|      \|      \|
+-+--------+--------+--------+----+   +----+--------+--------+--------+-+
|                                 |   |                                 |
|              A                  |   |               B                 |
|                                 |   |                                 |
+-+--------+--------+--------+----+   +----+--------+--------+--------+-+
  |\      |\      |\      |\           /|      /|      /|      /|
 ......................................................................

  |    /   |    /   |    /   |    /   \    |   \    |   \    |   \    |
  |   /    |   /    |   /    |   /     \   |    \   |    \   |    \   |
  V  V     V  V     V  V     V  V       V  V     V  V     V  V     V  V
+-+--+-+ +-+--+-+ +-+--+-+ +-+--+-+   +-+--+-+ +-+--+-+ +-+--+-+ +-+--+-+
|      | |      | |      | |      |   |      | |      | |      | |      |
|      | |      | |      | |      |   |      | |      | |      | |      |
+-+--+-+ +-+--+-+ +-+--+-+ +-+--+-+   +-+--+-+ +-+--+-+ +-+--+-+ +-+--+-+
  |        |        |        |             |        |        |        |
  V        V        V        V             V        V        V        V
  1        2        3        4             5        6        7        8
```

fig.c)

d) Estimate the upper and lower bound for the number of the element switches in the n-universal scheme.

The 6-th competition -- Chelyabinsk, 1972.

form	first day				second day		
8	159	160	161		166	167	168
9	162a	163	161	164	169	170	171
10	162b	163	165	164	166	172	173

159.
Given a rectangle ABCD, points M -- the middle of [AD] side, N -- the middle of [BC] side. Let us take a point P on the continuation of the [DC] segment over the point D. Let us denote the point of intersection of lines (PM) and (AC) as Q.

Prove that the angles QNM and MNP are equal.

160.
Given 50 segments on the line. Prove that one of the following statements is valid.
1. Some 8 segments have the common point.
2. Some 8 segments do not intersect each other.

161.
Find the maximal x such that the expression $4^{27} + 4^{1000} + 4^x$ is the exact square.

162.
a) Let a,n,m be natural numbers, $a > 1$. Prove that if $(a^m + 1)$ is dividable by $(a^n + 1)$ than m is dividable by n.
b) Let a,b,n,m be natural numbers, $a > 1$, a and b are relatively prime. Prove that if $(a^m + b^m)$ is dividable by $(a^n + b^n)$ than m is dividable by n.

163.
```
       2
      / \
     /   \
    4     3
   / \   / \
 16  5  9   4
/ \ / \ /\ / \
```
The triangle table is built according to the rule: You put the natural number $a>1$ in the upper row, and then You write under the number k from the left side k^2, and from the right side -- $(k+1)$. For example, if $a = 2$, You get the table on the picture. Prove that all the numbers on each particular line are different.

164.
Given several squares with the total area 1. Prove that You can pose them in the square of the area 2 without any intersections.

165.
Let O be the intersection point of the of the convex quadrangle ABCD diagonals. Prove that the line drawn through the points of intersection of the medians of AOB and COD triangles is orthogonal to the line drawn through the points of intersection of the heights of BOC and AOD triangles.

166.
Each of the 9 straight lines divides the given square onto two quadrangles with the areas related as 2:3. Prove that there exist three of them intersecting in one point.

167.
The 7-angle $A_1A_2A_3A_4A_5A_6A_7$ is inscribed in a circle.

Prove that if the centre of the circle is inside the 7-angle, than the sum of A_1, A_2 and A_3 angles is less than 450 degrees.

168.
A game for two. One gives a digit and the second substitutes it instead of a star in the following difference:

    **** - **** =

Then the first gives the next digit, and so on 8 times.
The first wants to obtain the greatest possible difference, the second -- the least. Prove that:

1. The first can operate in such a way that the difference would be not less than 4000, not depending on the second's behaviour.
2. The second can operate in such a way that the difference would be not greater than 4000, not depending on the first's behaviour.

169.
Let x,y be positive numbers, s -- the least of $\{ x, (y+ 1/x), 1/y\}$. What is the greatest possible value of s? To what x and y does it correspond?

170.
The point O inside the convex polygon makes isosceles triangle with all the pairs of its vertices.
Prove that O is the centre of the outscribed circle.

171.
Is it possible to put the numbers 0,1 or 2 in the unit squares of the cross-lined paper 100x100 in such a way, that every rectangle 3x4 (and 4x3) would contain three zeros, four ones and five twos?

172.
Let the sum of positive numbers $x_1, x_2, \ldots , x_n$ be 1.
Let s be the greatest of the numbers

$$\frac{x_1}{1 + x_1}, \frac{x_2}{1 + x_1 + x_2}, \ldots , \frac{x_n}{1 + x_1 + \ldots + x_n}.$$

What is the minimal possible s? What x_i correspond it.
/* Derivatives were not included in the school plans in Russia. They expected another solution. -VAP */

173.
One-round hockey tournament is finished (each plays with each one time, the winner gets 2 points, looser -- 0, and 1 point for draw). For arbitrary subgroup of teams there exists a team (may be from that subgroup) that has got an odd number of points in the games with the teams of the subgroup.

Prove that there was even number of the participants.

The 7-th competition -- Kishenew, 1973.

form	first day			second day			
8	174a	175	176	182	183	184ab	
9	174b	177	178	179	185	186	184c
10	180	177	181	187	188	184c	

174.
Fourteen coins are submitted to the judge. An expert knows, that the coins from number one to seven are false, and from 8 to 14 -- normal. The judge is sure only that all the true coins have the same weight and all the false coins weights equal each other, but are less then the weight of the true coins. The expert has the scales without weights.

a) The expert wants to prove, that the coins 1--7 are false. How can he do it in three weighings?
b) How can he prove, that the coins 1--7 are false and the coins 8--14 are true in three weighings?

175.
Prove that 9-digit number, that contains all the decimal digits except zero and does not ends with 5 can not be exact square.

176.
Given n points, $n > 4$. Prove that You can connect them with arrows, in such a way, that You can reach every point from every other point, having passed through one or two arrows. (You can connect every pair with one arrow only, and move along the arrow in one direction only.)

177.
Given an angle with the vertex O and a circle touching its sides in the points A and B. A ray is drawn from the point A parallel to [OB). It intersects with the circumference in the point C. The segment [OC] intersects the circumference in the point E. The straight lines (AE) and (OB) intersect in the point K. Prove that $|OK| = |KB|$.

178.
The real numbers a,b,c satisfy the condition: for all x, such that $-1 \le x \le 1$, helds unequality $| ax^2 + bx + c | \le 1$.
Prove that for the same x helds $| cx^2 + bx + a | \le 2$.

179.
The tennis federation has assigned numbers to 1024 sportsmen, participating in the tournament, according to their skill. (The tennis

federation uses the olympic system of tournaments. The looser in the pair leaves, the winner meets with the winner of another pair. Thus, in the second tour remains 512 participants, in the third -- 256, et.c. The winner is determined after the tenth tour.) It comes out, that in the play between the sportsmen whose numbers differ more than on 2 always win that whose number is less.
What is the greatest possible number of the winner?

180.
The square polynomial $f(x) = ax^2 + bx + c$ is of such a sort, that the equation $f(x) = x$ does not have real roots.
Prove that the equation $f(f(x))$ does not have real roots also.

181.
n squares of the infinite cross-lined sheet of paper are painted with black colour (others are white). Every move all the squares of the sheet change their colour simultaneously. The square gets the colour, that had the majority of three ones: the square itself, its neighbour from the right side and its neighbour from the upper side.

a) Prove that after the finite number of the moves all the black squares will disappear.
b) Prove that it will happen not later than on the n-th move.

182.
Three similar acute-angled triangles AC_1B, BA_1C and CB_1A are built on the outer side of the acute-angled triangle ABC. (Equal triples of the angles are AB_1C, ABC_1, A_1BC and BA_1C, BAC_1, B_1AC.)

a) Prove that the circumferences outscribed around the outer triangles intersect in one point.
b) Prove that the straight lines AA_1, BB_1 and CC_1 intersect in the same point.

183.
N men are not acquainted each other. You need to introduce some of them to some of them in such a way, that all the men will have different number of the acquaintances. Prove that it is possible for all N.

184.
The king have revised the chess-board 8x8 having visited all the fields once only and returned to the starting point. When his trajectory was drawn (the centres of the squares were connected with the straight lines), a closed broken line without self-intersections appeared.

a) Give an example that the king could make 28 steps parallel the sides of the board only.
b) Prove that he could not make less than 28 such a steps.
c) What is the maximal and minimal length of the broken line

if the side of a field is 1?

185.
Given a triangle with a,b,c sides and with the area 1. a >= b >= c.
Prove that b^2 >= 2.

186.
Given a convex n-angle with pairwise (mutually) non-parallel sides
/* who knows Russian -- a letter "r" had broken on the
organising committee's typewriter - and it became
an inexhaustible source of jokes for some years */
and a point inside it. Prove that there are not more than n straight
lines coming through that point and halving the area of the n-angle.

187.
Prove that for every positive x_1, x_2, x_3, x_4, x_5 helds inequality:
(x_1 + x_2 + x_3 + x_4 + x_5)^2 >=
>= 4(x_1x_2 + x_3x_4 + x_5x_1 + x_2x_3 + x_4x_5).

188.
Given 4 points in three-dimensional space, not lying in one plane.
What is the number of such a parallelepipeds (bricks), that each point
is a vertex of each parallelepiped?

The 8-th competition -- Erevan, 1974.

form	first day					second day			
8	189abc	190	191			197	198	199	200a
9	190	192	189d	193		201	202	200b	
10	194	195	196	193		203	204	200b	

189.
a,b,c) Given some cards with either "-1" or "+1" written on the opposite
side. You are allowed to choose a triple of cards and ask about the
product of the three numbers on the cards. What is the minimal number
of questions allowing to determine all the numbers on the cards ...
a) for 30 cards,
b) for 31 cards,
c) for 32 cards.
(You should prove, that You cannot manage with less questions.)
d) Fifty abovementioned cards are lying along the circumference. You
are allowed to ask about the product of three consecutive numbers
only. You need to determine the product af all the 50 numbers.
What is the minimal number of questions allowing to determine it?

190.
Among all the numbers representable as ${36}^k - 5^l$ (k and l are natural numbers) find the smallest.
Prove that it is really the smallest.

191.
a) Each of the side of the convex hexagon (6-angle) is longer than 1. Does it necessary have a diagonal longer than 2?
b) Each of the main diagonals of the convex hexagon is longer than 2. Does it necessary have a side longer than 1?

192.
Given two circles with the radiuses R and r, touching each other from the outer side. Consider all the trapezoids, such that its lateral sides touch both circles, and its bases touch different circles.
Find the shortest possible lateral side.

193.
Given n vectors of unit length in the plane. The length of their total sum is less than one. Prove that You can rearrange them to provide the property: for every k, $k \le n$, the length of the sum of the first k vectors is less than 2.

194.
Find all the real a,b,c such that the equality
$|ax + by + cz| + |bx + cy + az| + |cx + ay + bz| = |x| + |y| + |z|$
is valid for all the real x,y,z.

195.
Given a square $ABCD$. Points P and Q are in the sides $[AB]$ and $[BC]$ respectively. $|BP| = |BQ|$. Let H be the base of the perpendicular from the point B to the segment $[PC]$.
Prove that the angle DHQ is a right one.

196.
Given some red and blue points. Some of them are connected by the segments. Let us call "exclusive" the point, if its colour differs from the colour of more than half of the connected points. Every move one arbitrary "exclusive" point is repainted to the other colour.
Prove that after the finite number of moves there will remain no "exclusive" points.

197.
Find all the natural n and k such that n^n has k digits and k^k has n digits.

198.
Given points D and E on the legs [CA] and [CB], respectively, of the isosceles right triangle. $|CD| = |CE|$. The extensions of the perpendiculars from D and C to the line AE cross the hypotenuse AB in the points K and L. Prove that $|KL| = |LB|$.

199.
Two are playing the game "cats and rats" on the chess-board 8x8. The first has one piece -- a rat, the second -- several pieces -- cats. All the pieces have four available moves -- up, down, left, right -- to the neighbour field, but the rat can also escape from the board if it is on the boarder of the chess-board. If they appear on the same field -- the rat is eaten. The players move in turn, but the second can move all the cats in independent directions.

a) Let there be two cats. The rat is on the interior field. Is it possible to put the cats on such a fields on the border that they will be able to catch the rat?
b) Let there be three cats, but the rat moves twice diring the first turn. Prove that the rat can escape.

200.
a) Prove that You can rearrange the numbers 1, 2, ... , 32 in such a way, that for every couple of numbers none of the numbers between them will equal their arithmetic mean.
b) Can You rearrange the numbers 1, 2, ... , 100 in such a way, that for every couple of numbers none of the numbers between them will equal their arithmetic mean?

201.
Find all the three-digit numbers such that it equals to the arithmetic mean of the six numbers obtained by rearranging its digits.

202.
Given a convex polygon. You can put no triangle with area 1 inside it. Prove that You can put the polygon inside a triangle with the area 4.

203.
Given a function $f(x)$ on the segment $0 \le x \le 1$. For all x, $f(x) \ge 0$; $f(1) = 1$. For all the couples of x_1, x_2 such, that all the arguments are in the segment $f(x_1 + x_2) \ge f(x_1) + f(x_2)$.

a) Prove that for all x helds $f(x) \le 2x$.
b) Is the inequality $f(x) \le 1.9x$ valid?

204.
Given a triangle ABC with the are 1. Let A',B' and C' are the middles of the sides [BC], [CA] and [AB] respectively.

What is the minimal possible area of the common part of two triangles A'B'C' and KLM, if the points K,L and M are lying on the segments [AB'], [CA'] and [BC'] respectively?

The 9-th competition -- Saratov, 1975.

form	first day				second day		
8	205a	206	207	208a	213	214	215
9	209	206	210	208b	216	215	217
10	211	212	205b	208	214	218	219

205.

a) The triangle ABC was turned around the centre of the outscribed circle by the angle less than 180 degrees and thus was obtained the triangle $A_1B_1C_1$. The corresponding segments [AB] and $[A_1B_1]$ intersect in the point C_2; [BC] and $[B_1C_1]$ -- A_2; [AC] and $[A_1C_1]$ -- B_2.
Prove that the triangle $A_2B_2C_2$ is similar to the triangle ABC.

b) The quadrangle ABCD was turned around the centre of the outscribed circle by the angle less than 180 degrees and thus was obtained the quadrangle $A_1B_1C_1D_1$.
Prove that the points of intersection of the corresponding lines ((AB) and (A_1B_1), (BC) and (B_1C_1), (CD) and (C_1D_1), (DA) and (D_1A_1)) are the vertices of the parallelogram.

206.

Given a triangle ABC with the unit area. The first player chooses a point X on the side [AB], than the second -- Y on [BC] side, and, finally, the first chooses a point Z on [AC] side. The first tries to obtain the greatest possible area of the XYZ triangle, the second -- the smallest. What area can obtain the first for sure and how?

207.

What is the smallest perimeter of the convex 32-angle, having all the vertices in the nodes of cross-lined paper with the sides of its squares equal to 1?

208.

a) Given a big square consisting of 7x7 squares. You should mark the centres of k points in such a way, that no quadruple of the marked points will be the vertices of a rectangle with the sides parallel to the sides of the given squares.
What is the greatest k such that the problem has solution?

b) The same problem for 13×13 square.

209.
Denote the middles of the convex hexagon $A_1A_2A_3A_4A_5A_6$ diagonals A_6A_2, A_1A_3, A_2A_4, A_3A_5, A_4A_6, A_5A_1 as B_1, B_2, B_3, B_4, B_5, B_6 respectively. Prove that if the hexagon $B_1B_2B_3B_4B_5B_6$ is convex, than its area equals to the quarter of the initial hexagon.

210.
Prove that it is possible to find 2^{n+1} of 2^n digit numbers containing only "1" and "2" as digits, such that every two of them distinguish at least in 2^{n-1} digits.

211.
Given a finite set of polygons in the plane.
Every two of them have a common point.
Prove that there exists a straight line, that crosses all the polygons.

212.
Prove that for all the positive numbers a,b,c the following unequality is valid:
$a^3 + b^3 + c^3 + 3abc > ab(a + b) + bc(b + c) + ac(a + c)$.

213.
Three flies are crawling along the perimeter of the ABC triangle in such a way, that the centre of their masses is a constant point. One of the flies has already passed along all the perimeter.
Prove that the centre of the flies' masses coincides with the centre of masses of the ABC triangle. (The centre of masses for the triangle is the point of medians intersection.)

214.
Several zeros, ones and twos are written on the blackboard. An anonymous clean in turn pairs of different numbers, writing, instead of cleaned, the number not equal to each. (0 instead of pair {1,2}; 1 instead of {0,2}; 2 instead of {0,1}). Prove that if there remains one number only, it does not depend on the processing order.

215.
Given a horizontal strip on the plane (its sides are parallel lines) and n lines intersecting the strip. Every two of them intersect inside the strip, and not a triple has a common point. Consider all the paths along the segments of those lines, starting on the lower side of the strip and ending on the upper side with the properties: moving along such a path we are constantly rising up, and, having reached the intersection, we are obliged to turn to another line. Prove that:
a) there are not less than $n/2$ such a paths without common points;
b) there is a path consisting of not less than of n segments;

c) there is a path that goes along not more than along $n/2+1$ lines;
d) there is a path that goes along all the n lines.

216.
For what k is it possible to construct a cube $k \times k \times k$ of the black and white cubes $1 \times 1 \times 1$ in such a way that every small cube has the same colour, that have exactly two his neighbours. (Two cubes are neighbours, if they have the common face.)

217.
Given a polynomial $P(x)$ with
a) natural coefficients;
b) integer coefficients;
Let us denote with a_n the sum of the digits of $P(n)$ value.
Prove that there is a number encountered in the sequence
$a_1, a_2, \ldots, a_n, \ldots$ infinite times.

218.
The world and the european champion are determined in the same tournament carried in one round. There are 20 teams and k of them are european. The european champion is determined according to the results of the games only between those k teams. What is the greatest k such that the situation, when the single european champion is the single world outsider, is possible if:
a) it is hockey (draws allowed)?
b) it is volleyball (no draws)?

219.
a) Given real numbers a_1, a_2, b_1, b_2 and positive p_1, p_2, q_1, q_2. Prove that in the table 2x2

$$\begin{matrix} \frac{a_1 + b_1}{p_1 + q_1} & \frac{a_1 + b_2}{p_1 + q_2} \\ \\ \frac{a_2 + b_1}{p_2 + q_1} & \frac{a_2 + b_2}{p_2 + q_2} \end{matrix}$$

there is a number in the table, that is not less than another number in the same row and is not greater than another number in the same column (a saddle point).
b) Given real numbers $a_1, a_2, \ldots, a_n$; $b_1, b_2, \ldots, b_n$ and positive $p_1, p_2, \ldots, p_n$; $q_1, q_2, \ldots, q_n$.
We build the table $n \times n$, with the numbers $(0 < i,j \le n)$

$$\frac{a_i + b_j}{p_i + q_j}$$

in the intersection of the i-th row and j-th column.
Prove that there is a number in the table, that is not less than arbitrary number in the same row and is not greater than arbitrary number in the same column (a saddle point).

The 10-th competition -- Dushanbe, 1976.

form	first day					second day		
8	220	221	222ab	223		229	230	231
9	222b	224	223	225		230	232	231
10	223	226	227	228	225	233	234	231

220.
There are 50 exact watches lying on a table. Prove that there exist a certain moment, when the sum of the distances from the centre of the table to the ends of the minute hands is more than the sum of the distances from the centre of the table to the centres of the watches.

221.
A row of 1000 numbers is written on the blackboard. We write a new row, below the first according to the rule: We write under every number a the natural number, indicating how many times the number a is encountered in the first line. Then we write down the third line: under every number b -- the natural number, indicating how many times the number b is encountered in the second line, and so on.

a) Prove that there is a line that coincides with the preceding one.
b) Prove that the eleventh line coincides with the twelfth.
c) Give an example of the initial line such, that the tenth row differs from the eleventh.

222.
Given three circumferences of the same radius in a plane.
a) All three are crossing in one point K. Consider three arcs AK,CK,EK : the A,C,E are the points of the circumferences intersection and the arcs are taken in the clockwise direction. (Sorry, no picture. Every arc is inside one circle, outside the second and on the border of the third one) Prove that the sum of the arcs is 180 degrees.
b) Consider the case, when the three circles give a curvilinear triangle BDF as there intersection (instead of one point K). Prove that the sum of the AB, CD and EF arcs is 180 degrees.

(The arcs are taken in the clockwise direction. Every arc is inside one circle, outside the second and on the border of the third one)

223.
The natural numbers x_1 and x_2 are less than 1000. We build a sequence:
$x_3 = |x_1 - x_2|$;
$x_4 = \min \{ |x_1 - x_2|, |x_1 - x_3|, |x_2 - x_3|\}$;
....
$x_k = \min \{ |x_i - x_j|; 0 < i < j < k\}$;
....
Prove that $x_{21} = 0$.

224.
Can You mark the cube's vertices with the three-digit binary numbers in such a way, that the numbers at all the possible couples of neighbouring vertices differ in at least two digits?

225.
Given 4 vectors a,b,c,d in the plane, such that $a + b + c + d = 0$.
Prove that helds inequality:
$|a| + |b| + |c| + |d| >= |a + d| + |b + d| + |c + d|$.

226.
Given right 1976-angle. The middles of all the sides and diagonals are marked. What is the greatest number of the marked points lying on one circumference?

227.
There are n rectangles drawn on the rectangular sheet of paper with the sides of the rectangles parallel to the sheet sides. The rectangles do not have pairwise common interior points. Prove that after cutting out the rectangles the sheet will split into not more than n+1 part.

228.
There are three straight roads. Three pedestrians are moving along those roads, and they are NOT on one line in the initial moment. Prove that they will be one line not more than twice.

229.
Given a chess-board 99x99 with a set F of fields marked on it (the set is different in three tasks). There is a beetle sitting on every field of the set F. Suddenly all the beetles have raised into the air and flied to another fields of the same set. The beetles from the neighbouring fields have landed either on the same field or on the neighbouring ones (may be far from their starting point). (We consider the fields to be neighbouring if they have at least one common vertex.)

Consider a statement: "There is a beetle, that either stayed on the same field or moved to the neighbouring one". Is it always valid if the figure F is:

a) A central cross, i.e. the union of the 50-th row and the 50-th column?
b) A window frame, i.e. the union of the 1-st, 50-th and 99-th rows and the 1-st, 50-th and 99-th columns?
c) All the chess-board?

230.
Let us call "big" a triangle with all sides longer than 1. Given a equilateral triangle with all the sides equal to 5. Prove that:

a) You can cut 100 big triangles out of given one.
b) You can divide the given triangle onto 100 big nonintersecting ones fully covering the initial one.
c) The same as b), but the triangles either do not have common points, or have one common side, or one common vertex.
d) The same as c), but the initial triangle has the side 3.

231.
Given natural n. We shall call "universal" such a sequence of natural number $a_1, a_2, \ldots, a_k$; $k>=n$, if we can obtain every transposition of the first n natural numbers (i.e such a sequence of n numbers, that every one is encountered only once) by deleting some its members. (Examples: (1,2,3,1,2,1,3) is universal for n=3, and (1,2,3,2,1,3,1) -- not, because You can't obtain (3,1,2) from it.) The goal is to estimate the length of the shortest universal sequence for given n.

a) Give an example of the universal sequence of n^2 members.
b) Give an example of the universal sequence of $(n^2 -n + 1)$ members.
c) Prove that every universal sequence contains not less than $n(n + 1)/2$ members
d) Prove that the shortest universal sequence for n=4 contains 12 members
e) Find as short universal sequence, as You can. The Organising Committee knows the method for $(n^2 - 2n +4)$ members.

232.
n numbers are written down along the circumference. Their sum equals to zero, and one of them equals 1.

a) Prove that there are two neighbours with their difference not less than n/4.
b) Prove that there is a number that differs from the arithmetic mean of its two neighbours not less than on $8/{n^2}$.
c) Try to improve the previous estimation, i.e what number can be used instead of 8?

d) Prove that for $n=30$ there is a number that differs from the arithmetic mean of its two neighbours not less than on $2/113$; give an example of such 30 numbers along the circumference, that not a single number differs from the arithmetic mean of its two neighbours more than on $2/113$.

233.
Given right n-angle wit the point O -- its centre. All the vertices are marked either with +1 or -1. We may change all the signs in the vertices of right k-angle ($2 \le k \le n$) with the same centre O. (By 2-angle we understand a segment, being halved by O.)
Prove that in a), b) and c) cases there exists such a set of (+1)s and (-1)s, that we can never obtain a set of (+1)s only.
a) $n = 15$;
b) $n = 30$;
c) $n > 2$;
d) Let us denote $K(n)$ the maximal number of (+1) and (-1) sets such, that it is impossible to obtain one set from another. Prove, for example, that $K(200) = 2^{80}$.

234.
Given a sphere of unit radius with the big circle (i.e of unit radius) that will be called "equator". We shall use the words "pole", "parallel","meridian" as self-explanatory.
a) Let $g(x)$, where x is a point on the sphere, be the distance from this point to the equator plane. Prove that $g(x)$ has the property
if x_1, x_2, x_3 are the ends of the pairwise orthogonal radiuses, than $g(x_1)^2 + g(x_2)^2 + g(x_3)^2 = 1$. (*)
Let function $f(x)$ be an arbitrary nonnegative function on a sphere that satisfies (*) property.
b) Let x_1 and x_2 points be on the same meridian between the north pole and equator, and x_1 is closer to the pole than x_2. Prove that $f(x_1) > f(x_2)$.
c) Let y_1 be closer to the pole than y_2. Prove that $f(y_1) > f(y_2)$.
d) Let z_1 and z_2 be on the same parallel. Prove that $f(z_1) = f(z_2)$.
e) Prove that for all x $f(x) = g(x)$.

The 11-th competition -- Tallinn, 1977.

form	first day					second day			
8	235	236	237b	238		243	244ab	245	246
9	237a	239	235	240		247	248	249	250
10	237a	239	241	242	235	251	244	246	

235.
Given a closed broken line without self-intersections in a plane. Not a triple of its vertices belongs to one straight line. Let us call "special" a couple of line's segments if the one's continuation intersects another. Prove that there is even number of special pairs.

236.
Given several points, not all lying on one straight line. Some number is assigned to every point. It is known, that if a straight line contains two or more points, than the sum of the assigned to those points equals zero. Prove that all the numbers equal to zero.

237.
a) Given a circle with two inscribed triangles T_1 and T_2. The vertices of T_1 are the middles of the arcs with the ends in the vertices of T_2. Consider a hexagon -- the intersection of T_1 and T_2. Prove that its main diagonals are parallel to T_1 sides and are intersecting in one point.
b) The segment, that connects the middles of the arcs AB and AC of the circle outscribed around the ABC triangle, intersects [AB] and [AC] sides in D and K points. Prove that the points A,D,K and O -- the centre of the circle -- are the vertices of a diamond.

238.
Several black an white checkers (tokens?) are standing along the circumference. Two men remove checkers in turn. The first removes all the black ones that had at least one white neighbour, and the second -- all the white ones that had at least one black neighbour. They stop when all the checkers are of the same colour.
a) Let there be 40 checkers initially. Is it possible that after two moves of each man there will remain only one (checker)?
b) Let there be 1000 checkers initially. What is the minimal possible number of moves to reach the position when there will remain only one (checker)?

239.
Given infinite sequence a_n. It is known that the limit of
$b_n = a_{n+1} - {a_n}/2$ equals zero.
Prove that the limit of a_n equals zero.

240.
There are direct routes from every city of a certain country to every other city. The prices are known in advance. Two tourists (they do not necessary start from one city) have decided to visit all the cities, using only direct travel lines. The first always chooses the cheapest

ticket to the city, he has never been before (if there are several -- he chooses arbitrary destination among the cheapests). The second -- the most expensive (they do not return to the first city). Prove that the first will spend not more money for the tickets, than the second. /* The fact seems to be evident, but the proof is not easy -- VAP */

241.
Every vertex of a convex polyhedron belongs to three edges. It is possible to outscribe a circle around all its faces. Prove that the polyhedron can be inscribed in a sphere.

242.
The polynomial $x^{10} + ?x^9 + ?x^8 + ... +?x + 1$ is written on the blackboard. Two players substitute (real) numbers instead of one of the question marks in turn. (9 turns total.) The first wins if the polynomial will have no real roots. Who wins?

243.
Seven elves are sitting at a round table. Each elf has a cup. Some cups are filled with some milk. Each elf in turn and clockwise divides all his milk between six other cups. After the seventh has done this, every cup was containing the initial amount of milk. How much milk did every cup contain, if there was three litres of milk total?

244.
Let us call "fine" the 2n-digit number if it is exact square itself and the two numbers represented by its first n digits (first digit may not be zero) and last n digits (first digit may be zero, but it may not be zero itself) are exact squares also.
a) Find all two- and four-digit fine numbers.
b) Is there any six-digit fine number?
c) Prove that there exists 20-digit fine number.
d) Prove that there exist at least ten 100-digit fine numbers.
e) Prove that there exists 30-digit fine number.

245.
Given a set of n positive numbers. For each its nonempty subset consider the sum of all the subset's numbers.
Prove that You can divide those sums onto n groups in such a way, that the least sum in every group is not less than a half of the greatest sum in the same group.

246.
There are 1000 tickets with the numbers 000, 001, ... , 999; and 100 boxes with the numbers 00, 01, ... , 99. You may put a ticket in a box, if You can obtain the box number from the ticket number by deleting one

digit. Prove that:

a) You can put all the tickets in 50 boxes;

b) 40 boxes is not enough for that;

c) it is impossible to use less than 50 boxes.

d) Consider 10000 4-digit tickets, and You are allowed to delete two digits. Prove that 34 boxes is enough for storing all the tickets.

e) What is the minimal used boxes set in the case of k-digit tickets?

247.

Given a square 100x100 on the sheet of cross-lined paper. There are several broken lines drawn inside the square. Their links consist of the small squares sides. They are neither pairwise- nor self-intersecting (have no common points). Their ends are on the big square boarder, and all the other vertices are in the big square interior. Prove that there exists (in addition to four big square angles) a node (corresponding to the cross-lining family, inside the big square or on its side) that does not belong to any broken line.

248.

Given natural numbers $x_1, x_2, \ldots, x_n$; $y_1, y_2, \ldots, y_m$.
The following condition is valid:
$(x_1 + x_2 + \ldots + x_n) = (y_1 + y_2 + \ldots + y_m) < mn$. (*)
Prove that it is possible to delete some terms from (*) (not all and at least one) and to obtain another valid condition.

249.

Given 1000 squares on the plane with their sides parallel to the coordinate axes. Let M be the set of those squares centres.
Prove that You can mark some squares in such a way, that every point of M will be contained not less than in one and not more than in four marked squares.

250.

Given scales and a set of n different weights. We take weights in turn and add them on one of the scales sides. Let us denote "L" the scales state with the left side down, and "R" -- with the right side down.

a) Prove that You can arrange the weights in such an order, that we shall obtain the sequence LRLRLRLR... of the scales states. (That means that the state of the scales will be changed after putting every new weight.)

b) Prove that for every n-letter word containing R's and L's only You can arrange the weights in such an order, that the sequence of the scales states will be described by that word.

251.

Let us consider one variable polynomials with the senior coefficient equal to one. We shall say that two polynomials $P(x)$ and $Q(x)$ commute, if $P(Q(x))=Q(P(x))$ (i.e. we obtain the same polynomial, having collected the similar terms).

a) For every a find all Q such that the Q degree is not greater than three, and Q commutes with $(x^2 - a)$.
b) Let P be a square polynomial, and k is a natural number. Prove that there is not more than one commuting with P k-degree polynomial.
c) Find the 4-degree and 8-degree polynomials commuting with the given square polynomial P.
d) R and Q commute with the same square polynomial P. Prove that Q and R commute.
e) Prove that there exists a sequence $P_2, P_3, \ldots, P_n, \ldots$ (P_k is k-degree polynomial), such that $P_2(x) = x^2 - 2$, and all the polynomials in this infinite sequence pairwise commute.

The 12-th competition -- Tashkent, 1978.

form	first day				second day			
8	252	253	254	255ab	260	261	262	263
9	252	253	256	257	260	261	264	265
10	258	259	255cde	257	260	266	267	268

252.
Let a_n be the closest to sqrt(n) integer.
Find the sum $1/{a_1} + 1/{a_2} + \ldots + 1/{a_{1980}}$.

253.
Given a quadrangle ABCD and a point M inside it such that ABMD is a parallelogram. the angle CBM equals to CDM.
Prove that the angle ACD equals to BCM.

254.
Prove that there is no m such that $({1978}^m - 1)$ is dividable by $({1000}^m - 1)$.

255.
Given a finite set K_0 of points (in the plane or space). The sequence of sets $K_1, K_2, \ldots, K_n, \ldots$ is build according to the rule: we take all the points of K_i, add all the symmetric points with respect to all its points, and, thus obtain K_{i+1}.

a) Let K_0 consist of two points A and B with the distance 1 unit between them. For what n the set K_n contains the point that is 1000 units far from A?
b) Let K_0 consist of three points that are the vertices of the equilateral triangle with the unit square. Find the area of minimal

convex polygon containing K_n.

K_0 below is the set of the unit volume tetrahedron vertices.

c) How many faces contain the minimal convex polyhedron containing K_1?

d) What is the volume of the abovementioned polyhedron?

e) What is the volume of the minimal convex polyhedron containing K_n?

256.

Given two heaps of checkers. the bigger contains m checkers, the smaller -- n (m>n). Two players are taking checkers in turn from the arbitrary heap. The players are allowed to take from the heap a number of checkers (not zero) divisible by the number of checkers in another heap. The player that takes the last checker in any heap wins.

a) Prove that if $m > 2n$, than the first can always win.

b) Find all x such that if $m > xn$, than the first can always win.

257.

Prove that there exists such an infinite sequence $\{x_i\}$, that for all m and all k ($m<>k$) helds $|x_m - x_k| > 1/|m-k|$.

258.

Let $f(x) = x^2 + x + 1$. Prove that for every natural $m>1$ the numbers $m, f(m), f(f(m)), \ldots$ are relatively prime.

259.

Prove that there exists such a number A that You can inscribe 1978 different size squares in the plot of the function $y = A \sin(x)$. (The square is inscribed if all its vertices belong to the plot.)

260.

Given three automates that deal with the cards with the pairs of natural numbers. The first, having got the card with (a,b), produces new card with (a+1,b+1); the second, having got the card with (a,b), produces new card with (a/2,b/2), if both a and b are even and nothing in the opposite case; the third, having got the pair of cards with (a,b) and (b,c) produces new card with (a,c). All the automates return the initial cards also. Suppose there was (5,19) card initially. Is it possible to obtain

a) (1,50)?

b) (1,100)?

c) Suppose there was (a,b) card initially ($a<b$). We want to obtain (1,n) card. For what n is it possible?

261.

Given a circle with radius R and inscribed n-angle with area S. We mark one point on every side of the given polygon. Prove that the perimeter of the polygon with the vertices in the marked points is not less than 2S/R.

262.
The checker is standing on the corner field of a nxn chess-board. Each of two players moves it in turn to the neighbour (i.e. that has the common side) field. It is forbidden to move to the field, the checker has already visited. That who cannot make a move losts.

a) Prove that for even n the first can always win, and if n is odd, than the second can always win.
b) Who wins if the checker stands initially on the neighbour to the corner field?

263.
Given n nonintersecting segments in the plane. Not a pair of those belong to the same straight line. We want to add several segments, connecting the ends of given ones, to obtain one nonselfintersecting broken line. Is it always possible?

264.
Given $0 < a \le x_1 \le x_2 \le \dots \le x_n \le b$. Prove that

$$(x_1 + x_2 + \dots + x_n)\left(\frac{1}{x_1} + \frac{1}{x_2} + \dots + \frac{1}{x_n}\right) \le \frac{(a+b)^2}{4ab}n^2.$$

265.
Given a simple number $p>3$. Consider the set M of the pairs (x,y) with the integer coordinates in the plane such that $0 \le x < p$; $0 \le y < p$. Prove that it is possible to mark p points of M such that not a triple of marked points will belong to one line and there will be no parallelogram with the vertices in the marked points.

266.
Prove that for every tetrahedron there exist two planes such that the projection areas on those planes relation is not less than $\sqrt{2}$.

267.
Given $a_1, a_2, \dots, a_n$. Define $b_k = (a_1 + a_2 + \dots + a_k)/k$ for $1 \le k \le n$. Let

$C = (a_1 - b_1)^2 + (a_2 - b_2)^2 + \dots + (a_n - b_n)^2$;

$D = (a_1 - b_n)^2 + (a_2 - b_n)^2 + \dots + (a_n - b_n)^2$.

Prove that C <= <D <= 2C.

268.
Consider a sequence $x_n = (1 + \sqrt{2} + \sqrt{3})^n$.
Each member can be represented as

$x_n = q_n + r_n\sqrt{2} + s_n\sqrt{3} + t_n\sqrt{6}$, where q_n, r_n, s_n, t_n are integers.

Find the limits of the fractions $\frac{r_n}{q_n}$, $\frac{s_n}{q_n}$, $\frac{t_n}{q_n}$.

The 13-th competition -- Tbilisi, 1979.

form	first day			second day			
8	269	270	271	274	275	276	277
9	269	272	271	278	279	280	281
10	273	272	271	276	275	282	283

269.
What is the least possible relation of two isosceles triangles areas, if three vertices of the first one belong to three different sides of the second one?

270.
A grasshopper is hopping in the angle $x>=0$, $y>=0$ of the coordinate plane (that means that it cannot land in the point with negative coordinate). If it is in the point (x,y), it can either jump to the point $(x+1,y-1)$, or to the point $(x-5,y+7)$.
Draw a set of such an initial points (x,y), that having started from there, a grasshopper cannot reach any point farther than 1000 from the point $(0,0)$. Find its area.

271.
Every member of a certain parliament has not more than 3 enemies. Prove that it is possible to divide it onto two subparliaments so, that everyone will have not more than one enemy in his subparliament. (A is the enemy of B if and only if B is the enemy of A.)

272.
Some numbers are written in the notebook. We can add to that list the arithmetic mean of some of them, if it doesn't equal to the number, already having been included in it. Let us start with two numbers, 0 and 1. Prove that it is possible to obtain
a) 1/5;
b) an arbitrary rational number between 0 and 1.

273.
For every n the decreasing sequence $\{x_k\}$ satisfies a condition

$$x_1 + \frac{x_4}{2} + \frac{x_9}{3} + \dots + \frac{x_{n^2}}{n} <= 1.$$

Prove that for every n it also satisfy

$$x_1 + \frac{x_2}{2} + \frac{x_3}{3} + \dots + \frac{x_n}{n} \le 3.$$

274.
Given some points in the plane. For some pairs A,B the vector AB is chosen. For every point the number of the chosen vectors starting in that point equal to the number of the chosen vectors ending in that point. Prove that the sum of the chosen vectors equals to zero vector.

275.
What is the least possible number of the checkers being required
a) for the 8x8 chess-board;
b) for the nxn chess-board;
to provide the property: Every line (of the chess-board fields) parallel to the side or diagonal is occupied by at least one checker?

276.
Find x and y (a and b parameters):

$$\frac{x - y\sqrt{x^2 - y^2}}{\sqrt{1 - x^2 + y^2}} = a; \quad \frac{y - x\sqrt{x^2 - y^2}}{\sqrt{1 - x^2 + y^2}} = b.$$

277.
Given some square carpets with the total area 4.
Prove that they can fully cover the unit square.

278.
Prove that for the arbitrary numbers $x_1, x_2, \dots, x_n$ from the [0,1] segment $(x_1 + x_2 + \dots + x_n + 1)^2 \ge$

$$\ge 4({x_1}^2 + {x_2}^2 + \dots + {x_n}^2).$$

279.
Natural p and q are relatively prime. The [0,1] is divided onto (p+q) equal segments.
Prove that every segment except two marginal contain exactly one from the (p+q-2) numbers {1/p, 2/p, ... , (p-1)/p, 1/q, 2/q, ... , (q-1q)}.

280.
Given the point O in the space and 1979 straight lines $l_1, l_2, \dots, l_{1979}$ containing it. Not a pair of lines is orthogonal.
Given a point A_1 on l_1 that doesn't coincide with O.
Prove that it is possible to choose the points A_i on l_i (i = 2, 3, ... , 1979) in so that 1979 pairs will be orthogonal:
A_1A_3 and l_2;

A_2A_4 and l_3;

....

$A_{i-1}A_{i+1}$ and l_i;

....

$A_{1977}A_{1979}$ and l_{1978};
$A_{1978}A_1$ and l_{1979};
$A_{1979}A_2$ and l_1

281.
The finite sequence $a_1, a_2, \ldots , a_n$ of ones and zeroes should satisfy a condition: for every k from 0 to (n-1) the sum $a_1a_{k+1} + a_2a_{k+2} + \ldots + a_{n-k}a_n$ should be odd.
a) Build such a sequence for n=25.
b) Prove that there exists such a sequence for some n > 1000.

282.
The convex quadrangle is divided by its diagonals onto four triangles. The circles inscribed in those triangles are equal. Prove that the given quadrangle is a diamond.

283.
Given n points (in sequence) $A_1, A_2, \ldots , A_n$ on a line. All the segments A_1A_2, A_2,A_3, ... $A_{n-1}A_n$ are shorter than 1. We need to mark (k-1) points so that the difference of every two segments, with the ends in the marked ponts, is shorter than 1.
Prove that it is possible
a) for k=3;
b) for every k less than (n-1).

The 14-th competition -- Saratov, 1980.

form	first day				second day			
8	284	285	286	287	293	294	295	296
9	288	289	286	290	295	297	298	299
10	291	289	292	290	300	301	302	303

284.
All the two-digit numbers from 19 to 80 are written in a line without spaces. Is the obtained number (192021....7980) dividable by 1980?

285.
The vertical side of a square is divided onto n segments. The sum of the segments with even numbers lengths equals to the sum of the segments with odd numbers lengths. (n-1) lines parallel to the horizontal sides are drawn from the segments ends, and, thus, n strips

are obtained. The diagonal is drawn from the lower left corner to the upper right one. This diagonal divides every strip onto left and right parts. Prove that the sum of the left parts of odd strips areas equals to the sum of the right parts of even strips areas.

286.
The load for the space station "Salut" is packed in containers. There are more than 35 containers, and the total weight is 18 metric tons. There are 7 one-way transport spaceships "Progress", each able to bring 3 metric tons to the station. It is known that they are able to take an arbitrary subset of 35 containers.
Prove that they are able to take all the load.

287.
The points M and P are the middles of [BC] and [CD] sides of a convex quadrangle ABCD. It is known that $|AM| + |AP| = a$.
Prove that the ABCD area is less than $\{a^2\}/2$.

288.
Are there three simple numbers x,y,z, such that $x^2 + y^3 = z^4$?

289.
Given a point E on the diameter AC of the certain circle.
Draw a chord BD to maximise the area of the quadrangle ABCD.

290.
There are several settlements on the bank of the Big Round Lake. Some of them are connected with the regular direct ship lines. Two settlements are connected if and only if two next (counterclockwise) to each ones are not connected.
Prove that You can move from the arbitrary settlement to another arbitrary settlement, having used not more than three ships.

291.
The six-digit decimal number contains six different non-zero digits and is dividable by 37. Prove that having transposed its digits You can obtain at least 23 more numbers dividable by 37.

292.
Find real solutions of the system
$\sin x + 2 \sin(x+y+z) = 0,$
$\sin y + 3 \sin(x+y+z) = 0,$
$\sin z + 4 \sin(x+y+z) = 0.$

293.
Given 1980 vectors in the plane, and there are some non-collinear among

them. The sum of every 1979 vectors is collinear to the vector not included in that sum.
Prove that the sum of all vectors equals to the zero vector.

294.
Let us denote with $S(n)$ the sum of all the digits of n.
a) Is there such an n that $n + S(n) = 1980$?
b) Prove that at least one of two arbitrary successive natural numbers is representable as $n + S(n)$ for some third number n.

295.
Some squares of the infinite sheet of cross-lined paper are red.
Each 2x3 rectangle (of 6 squares) contains exactly two red squares.
How many red squares can be in the 9x11 rectangle of 99 squares?

296.
An epidemic influenza broke out in the elves city. First day some of them were infected by the external source of infection and nobody later was infected by the external source. The elf is infected when visiting his ill friend. In spite of the situation every healthy elf visits all his ill friends every day. The elf is ill one day exactly, and has the immunity at least on the next day. There is no graftings in the city.
Prove that
a) If there were some elves immunised by the external source on the first day, the epidemic influenza can continue arbitrary long time.
b) If nobody had the immunity on the first day, the epidemic influenza will stop some day.

297.
Let us denote with $P(n)$ the product of all the digits of n. Consider the sequence $n_{k+1} = n_k + P(n_k)$. Can it be unbounded for some n_1?

298.
Given equilateral triangle ABC. Some line, parallel to [AC] crosses [AB] and [BC] in M and P points respectively. Let D be the centre of PMB triangle, E - the middle of the [AP] segment.
Find the angles of DEC triangle.

299.
Let the edges of rectangular parallelepiped be x,y and z ($x<y<z$), and $p=4(x+y+z)$, $s=2(xy+yz+zx)$ and $d=\sqrt{x^2 + y^2 + z^2}$ -- its perimeter, surface area and diagonal length, respectively. Prove that

$$x < \frac{1}{3}\left(\frac{p}{4} - \sqrt{d^2 - \frac{s}{2}}\right), \quad z > \frac{1}{3}\left(\frac{p}{4} + \sqrt{d^2 - \frac{s}{2}}\right).$$

300.
The A set consists of integers only. Its minimal element is 1 and its maximal element is 100. Every element of A except 1 equals to the sum of two (may be equal) numbers being contained in A. What is the least possible number of elements in A?

301.
Prove that there is an infinite number of such numbers B that the equation $[x^{3/2}] + [y^{3/2}] = B$ has at least 1980 integer solutions (x,y). ([z] denotes the greatest integer not exceeding z.)

302.
The edge [AC] of the tetrahedron ABCD is orthogonal to [BC], and [AD] is orthogonal to [BD]. Prove that the cosine of the angle between (AC) and (BD) lines is less than $|CD|/|AB|$.

303.
The number x from [0,1] is written as an infinite decimal fraction. Having rearranged its first five digits after the point we can obtain another fraction that corresponds to the number x_1. Having rearranged five digits of x_k from (k+1)-th till (k+5)-th after the point we obtain the number x_{k+1}.

a) Prove that the sequence x_i has limit.
b) Can this limit be irrational if we have started with the rational number?
c) Invent such a number, that always produces irrational numbers, no matter what digits were transposed.

The 15-th competition -- Alma-Ata, 1981.

form	first day				second day			
8	304	305	306	307	315	316	317	318
9	308	309	310	311	319	320	321	322
10	311	312	312	314	323	324	325	326

304.
Two equal chess-boards (8x8) have the same centre, but one is rotated by 45 degrees with respect to another. Find the total area of black fields intersection, if the fields have unit length sides.

305.
Given points A,B,M,N on the circumference. Two chords [MA_1] and [MA_2] are orthogonal to (NA) and (NB) lines respectively. Prove that (AA_1) and (BB_1) lines are parallel.

306.
Let us say, that a natural number has the property $P(k)$ if it can be represented as a product of k succeeding natural numbers greater than 1.

a) Find k such that there exists n which has properties $P(k)$ and $P(k+2)$ simultaneously.
b) Prove that there is no number having properties $P(2)$ and $P(4)$ simultaneously.

307.
The rectangular table has four rows. The first one contains arbitrary natural numbers (some of them may be equal). The consecutive lines are filled according to the rule: we look through the previous row from left to the certain number n and write the number k if n was met k times. Prove that the second row coincides with the fourth one.

308.
Given real a. Find the least possible area of the rectangle with the sides parallel to the coordinate axes and containing the figure determined by the system of inequalities
$y \le -x^2$, $y \ge x^2 - 2x + a$.
/* as there is no derivatives in the school program of the 9-th form, the participants had to use parabola properties -VAP */

309.
Three equilateral triangles ABC, CDE, EHK (the vertices are mentioned counterclockwise) are lying in the plane so, that the vectors [AD[and [DK[are equal. Prove that the triangle BHD is also equilateral.

310.
There are 1000 inhabitants in a settlement. Every evening every inhabitant tells all his friends all the news he had heard the previous day. Every news becomes finally known to every inhabitant. Prove that it is possible to choose 90 of inhabitants so, that if You tell them a news simultaneously, it will be known to everybody in 10 days.

311.
It is known about real a and b that the inequality
$a \cos(x) + b \cos(3x) > 1$ has no real solutions.
Prove that $|b| \le 1$.

312.
The points K and M are the centres of the AB and CD sides of the convex quadrangle ABCD. The points L and M belong to two other sides and KLMN is a rectangle. Prove that KLMN area is a half of ABCD area.

313.
Find all the sequences of natural k_n with two properties:
for all n $k_n \le n \sqrt{n}$;
for all $m>n$ $(k_n - k_m)$ is dividable by $(m-n)$.

314.
Is it possible to fill a rectangular table with black and white squares (only) so, that the number of black squares will equal to the number of white squares, and each row and each column will have more than 75% squares of the same colour?

315.
The quadrangles AMBE, AHBT, BKXM, and CKXP are parallelograms.
Prove that the quadrangle ABTE is also parallelogram.
(the vertices are mentioned counterclockwise)

316.
Find the natural solutions of the equation $x^3 - y^3 = xy + 61$.

317.
Eighteen soccer teams have played 8 tours of a one-round tournament.
Prove that there is a triple of teams, having not met each other yet.

318.
The points C_1, A_1, B_1 belong to [AB], [BC], [CA] sides, respectively, of the ABC triangle.

$$\frac{|AC_1|}{|C_1B|} = \frac{|BA_1|}{|A_1C|} = \frac{|CB_1|}{|B_1A|} = \frac{1}{3}.$$

Prove that the perimeter P of the ABC triangle and the perimeter p of the $A_1B_1C_1$ triangle satisfy inequality $P/2 < p < 3P/4$.

319.
Positive numbers x,y satisfy equality $x^3 + y^3 = x - y$.
Prove that $x^2 + y^2 < 1$.

320.
A pupil has tryed to make a copy of a convex polygon, drawn inside the unit circle. He draw one side, from its end -- another, and so on. Having finished, he has noticed that the first and the last vertices do not coincide, but are situated d units of length far from each other. The pupil draw angles precisely, but made relative error less than p in the lengths of sides. Prove that $d < 4p$.

321.
A number is written in the each vertex of a cube. It is allowed to add

one to two numbers written in the ends of one edge.
Is it possible to obtain the cube with all equal numbers if the numbers were initially as on the pictures:

```
  0-------0     7-------4     0-------1
 /|      /|    /|      /|    /|      /|
0-------0 |   6-------5 |   1-------0 |
| |     | |   | |     | |   | |     | |
| 0-----|-1   | 2-----|-3   | 0-----|-0
|/      |/    |/      |/    |/      |/
0-------0     1-------4     0-------0
  fig.a         fig.b         fig.c
```

322.
Find n such that each of the numbers n,(n+1),...,(n+20) has the common divider greater than one with the number 30030 = 2*3*5*7*11*13.

323.
The natural numbers from 100 to 999 are written on separate cards. They are gathered in one pile with their numbers down in arbitrary order. Let us open them in sequence and divide into 10 piles according to the least significant digit. The first pile will contain cards with 0 at the end, ... , the tenth -- with 9. Then we shall gather 10 piles in one pile, the first -- down, then the second, ... and the tenth -- up. Let us repeat the procedure twice more, but the next time we shall divide cards according to the second digit, and the last time -- to the most significant one.
What will be the order of the cards in the obtained pile?

324.
Six ponts are marked inside the 3x4 rectangle.
Prove that there is a pair of marked points with the distance between them not greater than sqrt(5).

325.
a) Find the minimal value of the polynomial
   P(x,y) = 4 + (x^2)(y^4) + (x^4)(y^2) - 3(x^2)(y^2)
   /* the usage of derivatives is forbidden, as they were
      not included in the school programs -- VAP */
b) Prove that it cannot be represented as a sum of the squares of some polynomials of x,y.

326.
The segments [AD], [BE] and [CF] are the side edges of the _right_ triangle prism.(I mean, that the equilateral triangle is a base -- VAP)
Find all the points in its base ABC, situated on the equal distances from the (AE), (BF) and (CD) lines.

The 16-th competition -- Odessa, 1982.

form	first day				second day			
8	327	328	329a	330	337	338	339	340
9	331	332	333	334	341	342	343	344
10	335	332	329b	336	345	346	347	348

327.
Given two points M and K on the circumference with radius r_1 and centre O_1. The circumference with radius r_2 and centre O_2 is inscribed in MO_1K angle. Find the MO_1KO_2 quadrangle area.

328.
Every member, starting from the third one, of two sequences $\{a_n\}$ and $\{b_n\}$ equals to the sum of two preceding ones. First members are: $a_1 = 1$, $a_2 = 2$, $b_1 = 2$, $b_2 = 1$. How many natural numbers are encountered in both sequences (may be on the different places)?

329.
a) Let m and n be natural numbers. For some nonnegative integers $k_1, k_2, \ldots, k_n$ the number $2^{k_1} + 2^{k_2} + \ldots + 2^{k_n}$ is dividable by $(2^m - 1)$. Prove that $n \geq m$.
b) Can You find a number, dividable by 111...1 (m times "1"), that has the sum of its digits less than m?

330.
A nonnegative real number is written at every cube's vertex. The sum of those numbers equals to 1. Two players choose in turn faces of the cube, but they cannot choose the face parallel to already chosen one (the first moves twice, the second -- once).
Prove that the firs player can provide the number, at the common for three chosen faces vertex, to be not greater than 1/6.

331.
Once upon a time, three boys visited a library for the first time.
The first decided to visit the library every second day.
The second decided to visit the library every third day.
The third decided to visit the library every fourth day.
The librarian noticed, that the library doesn't work on Wednesdays.
The boys decided to visit library on Thursdays, if they have to do it on Wednesdays, but to restart the day counting in these cases.
They strictly obeyed these rules. Some Monday later I met them all in that library.
What day of week was when they visited a library for the first time?

332.
The parallelogram ABCD isn't a diamond. The relation of the diagonal lengths |AC|/|BD| equals to k. The [AM) ray is symmetric to the [AD) ray with respect to the (AC) line. The [BM) ray is symmetric to the [BC) ray with respect to the (BD) line. (M point is those rays intersection.) Find the |AM|/|BM| relation.

333.
3k points are marked on the circumference. They divide it onto 3k arcs. Some k of them have length 1, other k of them have length 2, the rest k of them have length 3.
Prove that some two of the marked points are the ends of one diameter.

334.
Given a point M inside a right tetrahedron. Prove that at least one tetrahedron edge is seen from the M in an angle, that has a cosine not greater than -1/3. /*   I mean that if A and B are the vertices,
   corresponding to that edge, cos(AMB) <= -1/3 */

335.
Three numbers a,b,c belong to]0,pi/2[interval.
cos(a) = a;   sin(cos(b)) = b; cos(sin(c)) = c.
Sort those numbers in increasing order.

336.
The closed broken line M has odd number of vertices -- A_1, A_2, ... , A_{2n+1} in sequence. Let us denote with S(M) a new closed broken line with vertices B_1, B_2, ... ,B_{2n+1} -- the middles of the first line links: B_1 is the middle of [A_1A_2], ... , B_{2n+1} -- of [A_{2n+1}A_1]. Prove that in a sequence M_1=S(M), ... , M_k = S(M_{k-1}), ... there is a broken line, homothetic to the M.

337.
All the natural numbers from 1 to 1982 are gathered in an array in an arbitrary order in computer's memory. The program looks through all the sequent pairs (first and second, second and third,...) and exchanges numbers in the pair, if the number on the lower place is greater than another. Then the program repeats the process, but moves from another end of the array. The number, that stand initially on the 100-th place reserved its place. Find that number.

338.
Cucumber river in the Flower city /* authors mean the N.Nosov's tale about the sort of elves country */ has parallel banks with the distance between them 1 metre. It has some islands with the total perimeter 8

metres. Mr. Know-All claims that it is possible to cross the river in a boat from the arbitrary point, and the trajectory will not exceed 3 metres. Is he right?

339.
There is a parabola $y = x^2$ drawn on the coordinate plane. The axes are deleted. Can You restore them with the help of compass and ruler?

340.
The square table nxn is filled by integers. If the fields have common side, the difference of numbers in them doesn't exceed 1. Prove that some number is encountered not less than
a) not less than [n/2] times ([] mean the whole part);
b) not less than n times.

341.
Prove that the following inequality is valid for the positive x:
$2^{x^{1/12}} + 2^{x^{1/4}} \geq 2^{1 + x^{1/6}}$.

342.
What minimal number of numbers from the set {1,2,...,1982} should be deleted to provide the property: none of the remained numbers equals to the product of two other remained numbers?
/* This particular problem sounds bad due to the translator,
   but I had read the Moscow competition organising committee
   announcement with the word "prize" 7 times repeated in
   one sentence -- VAP */

343.
Every square on the infinite sheet of cross-lined paper contains some real number. Prove that some square contains a number that does not exceed at least four of eight neighbouring numbers.

344.
Given a sequence of real numbers $a_1, a_2, \ldots , a_n$. Prove that it is possible to choose some of the numbers providing 3 conditions:
a) not a triple of successive members is chosen;
b) at least one of every triple of successive members is chosen;
c) the absolute value of chosen numbers sum is not less that one sixth part of the initial numbers' absolute values sum.

345.
Given the square table nxn with (n-1) marked fields.
Prove that it is possible to move all the marked fields below the diagonal by moving rows and columns.

346.
Prove that the following inequality helds for all real a and natural n:
|a|*|a-1|*|a-2|*...*|a-n| >= F(a)*n!/(2^n),
F(a) is the distance from a to the closest integer.

347.
Can You find three polynomials P,Q,R of three variables x,y,z,
providing the condition:
a) P(x-y+z)^3 + Q(y-z-1)^3 +R(z-2x+1)^3 = 1;
b) P(x-y+z)^3 + Q(y-z-1)^3 +R(z-x+1)^3 = 1;
for all x,y,z?

348.
The KLMN tetrahedron (triangle pyramid) vertices are situated inside or
on the faces or on the edges of the ABCD tetrahedron. Prove that KLMN
perimeter is less than 4/3 ABCD perimeter.

The 17-th competition -- Kishenew, 1983.

form	first day				second day			
8	349	350	351	352	360	361	362	363
9	353	354	355	356	364	365	366	367
10	357	354	358	359	360	368	369	370

349.
Every cell of the net, drawn on the
picture has 1x1 size. Is it possible
to represent this net as a union of
the following sets:
a) Eight broken lines of length
   five each?
b) Five broken lines of length
   eight each?

```
+---+---+---+---+
|   |   |   |   |
+---+---+---+---+
|   |   |   |   |
+---+---+---+---+
|   |   |   |   |
+---+---+---+---+
|   |   |   |   |
+---+---+---+---+
```

350.
Three numbers were written with a chalk on the blackboard. The
following operation was repeated several times: One of the numbers was
cleared and the sum of two other numbers, decreased by 1, was written
instead of it. The final set of numbers is {17, 1967, 1983} /* In some
organisers opinion "17" is the most distinguishing number, and those
three numbers were met on the picturesque jubilee slogans. In fact, in
1967 the competition was just renamed -VAP */. Is it possible to admit
that the initial numbers were
a) {2, 2, 2}?

b) {3, 3, 3}?

351.
Three disks touch pairwise from outside in the points X,Y,Z. Then the radiuses of the disks were expanded by 2/sqrt(3) times, and the centres were reserved. Prove that the XYZ triangle is completely covered by the expanded disks.

353.
Find all the solutions of the system
y^2 = x^3 - 3x^2 + 2x,
x^2 = y^3 - 3y^2 + 2y.

354.
Natural number k has n digits in its decimal notation. It was rounded up to tens, then the obtained number was rounded up to hundreds, and so on (n-1) times. Prove that The obtained number m satisfies inequality m < 18*k/13. (Examples of rounding: 191->190->200; 135->140->100.)

355.
The point D is the middle of the [AB] side of the ABC triangle. The points E and F belong to [AC] and [BC] respectively.
Prove that the DEF triangle area does not exceed the sum of the ADE and BDF triangles areas.

356.
The sequences a_n and b_n members are the last digits of
[(sqrt(10))^n] and [(sqrt(2))^n] respectively (here [] denotes the whole part of a number). Are those sequences periodical?

357.
Two acute angles a and b satisfy condition
(sin(a))^2 + (sin(b))^2 = sin(a+b).
Prove that a + b = pi/2.

358.
The points A_1,B_1,C_1,D_1 and A_2,B_2,C_2,D_2 are orthogonal projections of the ABCD tetrahedron vertices on two planes.
Prove that it is possible to move one of the planes to provide the parallelness of (A_1A_2), (B_1B_2), (C_1C_2) and (D_1D_2) lines.

359.
The pupil is training in the square equation solution. Having the recurrent equation solved, he stops, if it doesn’t have two roots, or solves the next equation, with the free coefficient equal to the greatest root, the coefficient at x equal to the least root, and the

coefficient at x^2 equal to 1. Prove that the process cannot be infinite. What maximal number of the equations he will have to solve?

360.
Given natural n,m,k. It is known that m^n is dividable by n^m; and n^k is dividable by k^n. Prove that m^k is dividable by k^m.

361.
The Abba tribe language alphabet contains two letters only.
Not a word of this language is a beginning of another word.
Can this tribe vocabulary contain 3 four-letter, 10 five-letter,
30 six-letter and 5 seven-letter words?

362.
Can You fill the squares of the infinite cross-lined paper with integers so, that the sum of the numbers in every 4x6 fields rectangle would be
a) 10?
b) 1?

363.
The points A_1,B_1,C_1 belong to [BC],[CA],[AB] sides of the ABC triangle respectively. The $[AA_1]$, $[BB_1]$, $[CC_1]$ segments
split the ABC onto 4 smaller triangles and 3 quadrangles.
It is known, that the smaller triangles have the same area.
Prove that the quadrangles have equal areas. What is the
quadrangle area, it the small triangle has the unit area?

364.
The kindergarten group is standing in the column of pairs. The number of boys equals the number of girls in each of the two columns. The number of mixed (boy and girl) pairs equals to the number of the rest pairs. Prove that the total number of children in the group is dividable by eight.

365.
One side of the rectangle is 1cm. It is known that the rectangle can be divided by two orthogonal lines onto four rectangles, and each of the smaller rectangles has the area not less than 1 square centimetre, and one of them is not less than 2 square centimetres.
What is the least possible length of another side of big rectangle?

366.
Given a point O inside ABC triangle. Prove that
$S_A*[OA) + S_B*[OB) + S_C[OC) = [OO)$,
where S_A, S_B, S_C denote BOC, COA, AOB triangles areas respectively

([AB) denotes vector from A to B , [OO) -- zero vector).

367.
Given (2m+1) different integers, each absolute value is not greater than (2m-1). Prove that it is possible to choose three numbers among them, with their sum equal to zero.

368.
The points D,E,F belong to the sides]AB[,]BC[and]CA[of the ABC triangle respectively (but they are not vertices). Let us denote with d_0, d_1, d_2, and d_3 the maximal side length of the DEF, DEA, DBF, CEF, triangles respectively. Prove that
d_0 >= (sqrt(3)/2) min{d_1, d_2, d_3}. When the equality takes place?

369.
The M set consists of k non-intersecting segments on the line. It is possible to put an arbitrary segment shorter than 1cm on the line in such a way, that his ends will belong to M. Prove that the total sum of the segment lengths is not less than 1/k cm.

370.
The infinite decimal notation of the real number x contains all the digits. Let v_n be the number of different n-digit segments encountered in x notation. Prove that if for some n v_n <= (n+8), than x is a rational number.

The 18-th competition -- Ashkhabad, 1984.

form	first day				second day			
8	371	372	373	374	383	384	385	386
9	375	376	377	378	387	388	389	390
10	379	380	381	382	391	392	393	394

371.
a) The product of n integers equals n, and their sum is zero.
   Prove that n is dividable by 4.
b) Let n is dividable by 4.
   Prove that there exist n integers such, that their product equals n, and their sum is zero.

372.
Prove that every positive a and b satisfy inequality
((a+b)^2)/2 + (a+b)/4 >= a*sqrt(b) + b*sqrt(a).

373.

Given two equilateral triangles $A_1B_1C_1$ and $A_2B_2C_2$ in the plane. (The vertices are mentioned counterclockwise.) We draw vectors $[OA[$, $[OB[$, $[OC[$, from the arbitrary point O, equal to $[A_1A_2[$, $[B_1B_2[$, $[C_1C_2[$ respectively. Prove that the triangle ABC is equilateral.

374.
Given four colours and enough square plates 1x1. We have to paint four edges of every plate with four different colours and combine plates, putting them with the edges of the same colour together.
Describe all the pairs m,n, such that we can combine those plates in a nxm rectangle, that has every edge of one colour, and its four edges have different colours.

375.
Prove that every positive x,y and real a satisfy inequality
$x^{(\sin a)^2} \cdot y^{(\cos a)^2} < x + y$.

376.
Given a cube and two colours. Two players paint in turn a triple of arbitrary unpainted edges with his colour. (Everyone makes two moves.) The first wins if he has painted all the edges of an arbitrary face with his colour. Can he always win?

377.
n natural numbers ($n>3$) are written on the circumference. The relation of the two neighbours sum to the number itself is a whole number. Prove that the sum of those relations is
a) not less than $2n$;
b) less than $3n$.

378.
The circle with the centre O is inscribed in the ABC triangle. The circumference touches its sides [BC], [CA], [AB] in A_1, B_1, C_1 points respectively. The [AO], [BO], [CO] segments cross the circumference in A_2, B_2, C_2 points respectively.
Prove that (A_1A_2), (B_1B_2) and (C_1C_2) lines intersect in one point.

379.
Find integers m and n such that $(5 + 3\sqrt{2})^m = (3 + 5\sqrt{2})^n$.

380.
n real numbers are written in increasing order in a line. The same numbers are written in the second line below in unknown order. The third line contains the sums of the pairs of numbers above from two previous lines. It comes out, that the third line is arranged in increasing order.

Prove that the second line coincides with the first one.

381.
Given ABC triangle. From the P point three lines (PA),(PB),(PC) are drawn. They cross the outscribed circumference in A_1, B_1, C_1 points respectively. It comes out that the $A_1B_1C_1$ triangle equals to the initial one.
Prove that there are not more than eight such a points P in a plane.

382.
Positive x,y,z satisfy a system

$$x^2 + xy + (y^2)/3 = 25;$$
$$(y^2)/3 + z^2 = 9;$$
$$z^2 + zx + x^2 = 16.$$

Find the value of (xy + 2yz + 3zx) expression.

383.
The teacher wrote on a blackboard: " $x^2 + 10x + 20$ " Then all the pupils in the class came up in turn and either decreased or increased by 1 either the free coefficient or the coefficient at x, but not both.
Finally they have obtained: " $x^2 + 20x + 10$ ".
Is it true that some time during the process there was written the square polynomial with the integer roots?

384.
The centre of the coin with radius r is moved along some polygon with the perimeter P, that is outscribed around the circle with radius R (R>r). Find the coin trace area (a sort of polygon ring).

385.
There are scales and (n+1) weights with the total weight 2n. Each weight weight is an integer. We put all the weights in turn on the lighter side of the scales, starting from the heaviest one, and if the scales is in equilibrium -- on the left side. Prove that when all the weights will be put on the scales, they will be in equilibrium.

386.
Let us call "absolutely prime" the prime number, if having transposed its digits in an arbitrary order, we obtain prime number again. Prove that its notation cannot contain more than three different digits.

387.
The x and y figures satisfy a condition: for every n>=1 the number xx...x6yy...y4 (n times x and n times y) is an exact square.
Find all possible x and y.

388.
The A,B,C and D points (from left to right) belong to the straight line. Prove that every point E, that doesn't belong to the line satisfy:
$|AE| + |ED| + \big|\,|AB| - |CD|\,\big| > |BE| + |CE|$.

389.
Give a sequence $\{x_n\}$. $x_1 = x_2 = 1$. $x_{n+2} = (x_{n+1})^2 - x_n/2$.
Prove that the sequence has limit and find it.

390.
The white fields of 1983x1984 chess-board are filled with either +1 or -1. For every black field, the product of neighbouring numbers is +1. Prove that all the numbers are +1.

391.
The white fields of 3x3 chess-board are filled with either +1 or -1. For every field, let us calculate the product of neighbouring numbers. Then let us change all the numbers by the respective products. Prove that we shall obtain only +1's, having repeated this operation finite number of times.

392.
What is more 2/201 or ln(101/100)? (No differential calculus allowed).

393.
Given three circles c_1,c_2,c_3 with r_1,r_2,r_3 radiuses, $r_1 > r_2$; $r_1 > r_3$. Each lies outside of two others. The A point -- an intersection of the outer common tangents to c_1 and c_2 -- is outside c_3. The B point -- an intersection of the outer common tangents to c_1 and c_3 -- is outside c_2. Two pairs of tangents -- from A to c_3 and from B to c_2 -- are drawn. Prove that the quadrangle, they make, is outscribed around some circle and find its radius.

394.
Prove that every cube's cross-section, containing its centre, has the area not less then its face's area.

The 19-th competition -- Mogilev, 1985.

form	first day				second day			
8	395	396	397	398	407	408	409	410
9	399	400	401	402	411	410	412	413
10	403	404	405	406	414	415	416	417

395.

Two perpendiculars are drawn from the middles of each side of the acute-angle triangle to two other sides. Those six segments make hexagon. Prove that the hexagon area is a half of the triangle area.

396.
Is there any number n, such that the sum of its digits in the decimal notation is 1000, and the sum of its square digits in the decimal notation is 1000000?

397.
What maximal number of the dames (? Am I right? The checker turns into a dame, when it reaches the last row?) can be put on the chess-board 8x8 so, that every dame can be taken by at least one other dame?

398.
You should paint all the sides and diagonals of the right n-angle so, that every pair of segments, having the common point, would be painted with different colours. How many colours will You require?

399.
Given a straight line and the point O out of the line. Prove that it is possible to move an arbitrary point A in the same plane to the O point, using only rotations around O and symmetry with respect to the l.

400.
The senior coefficient a in the square polynomial $P(x) = ax^2 + bx + c$ is more than 100. What is the maximal number of integer values of x, such that $|P(x)|<50$?

401.
```
Different natural numbers a,b,...,k       k
are written in the diagram. Every         |
number in the diagram equals to           V
the sum of two numbers at the        h--->i
beginning of two arrows,             |    |
leading to the number.               V    V
What is the minimal             e--->f--->g
possible value of d?            |    |    |
                                V    V    V
                          a---->b--->c--->d
```
402.
Given unbounded strictly increasing sequence $a_1, a_2, \dots , a_n, \dots$ of positive numbers. Prove that
a) there exists a number k_0 such that for all $k>k_0$ the following inequality is valid:

$a_1/a_2 + a_2/a_3 + ... + a_k/a_{k-1} < k - 1$

b) there exists a number k_0 such that for all $k>k_0$ the following inequality is valid:

$a_1/a_2 + a_2/a_3 + ... + a_k/a_{k-1} < k - 1985$

403.

Find all the pairs (x,y) such that $|\sin(x)-\sin(y)| + \sin(x)\sin(y) <= 0$.

404.

The convex pentagon ABCDE _was_ drawn in the plane.
A_1 was symmetric to A with respect to B.
B_1 was symmetric to B with respect to C.
C_1 was symmetric to C with respect to D.
D_1 was symmetric to D with respect to E.
E_1 was symmetric to E with respect to A.
How is it possible to restore the initial pentagon with the compass and ruler, knowing A_1,B_1,C_1,D_1,E_1 points?

405.

The sequence $a_1, a_2, ... , a_k, ...$ is built according to the rules:
$a_{2n} = a_n$; $a_{4n+1} = 1$; $a_{4n+3} = 0$.
Prove that it is non-periodical sequence.

406.

n straight lines are drawn in a plane. They divide the plane onto several parts. Some of the parts are painted. Not a pair of painted parts has non-zero length common bound.
Prove that the number of painted parts is not more than $(n^2 + n)/3$.

407.

Given a cube, a cubic box, that exactly suits for the cube, and six colours. First man paints each side of the cube with its (side's) unique colour. Another man does the same with the box.
Prove that the third man can put the cube in the box in such a way, that every cube side will touch the box side of different colour.

408.

The $[A_0A_5]$ diameter divides a circumference with the O centre onto two hemicircumferences. One of them is divided onto five equal arcs A_0A_1, A_1A_2, A_2A_3, A_3A_4, A_4A_5. The (A_1A_4) line crosses (OA_2) and (OA_3) lines in M and N points.
Prove that $(|A_2A_3| + |MN|)$ equals to the circumference radius.

409.

If there are four numbers (a,b,c,d) in four registers of the calculating machine, they turn into $(a-b,b-c,c-d,d-a)$ numbers whenever

You press the button. Prove that if not all the initial numbers are equal, machine will obtain at least one number more than 1985 after some number of the operations.

410.
Numbers $1,2,3,\dots,2n$ are divided onto two equal groups. Let $a_1,a_2,\dots,a_n$ be the first group numbers in the increasing order, and $b_1,b_2,\dots,b_n$ -- the second group numbers in the decreasing order. Prove that $|a_1 - b_1| + |a_2 - b_2| + \dots + |a_n - b_n| = n^2$.

411.
The parallelepiped is constructed of the equal cubes. Three parallelepiped faces, having the common vertex are painted. Exactly half of all the cubes have at least one face painted. What is the total number of the cubes?

412.
One of two circumferences of radius R comes through A and B vertices of the ABCD parallelogram. Another comes through B and D. Let M be another point of circumferences intersection. Prove that the circle outscribed around AMD triangle has radius R.

413.
Given right hexagon. The lines parallel to all the sides are drawn from all the vertices and middles of the sides (consider only the interior, with respect to the hexagon, parts of those lines). Thus the hexagon is divided onto 24 triangles, and the figure has 19 nodes. 19 different numbers are written in those nodes. Prove that at least 7 of 24 triangles have the property: the numbers in its vertices increase (from the least to the greatest) counterclockwise.

414.
Solve the equation ("2" encounters 1985 times):

$$\cfrac{x}{2 + \cfrac{x}{2 + \cdots \cfrac{x}{2 + \cfrac{x}{1 + \sqrt{1 + x}}}}} = 1$$

415.
All the points situated more close than 1cm to ALL the vertices of

the right pentagon with 1cm side, are deleted from that pentagon. Find the area of the remained figure.

416.
Given big enough sheet of cross-lined paper with the side of the squares equal to 1. We are allowed to cut it along the lines only. Prove that for every $m>12$ we can cut out a rectangle of the greater than m area such, that it is impossible to cut out a rectangle of m area from it.

417.
The $ABCDA_1B_1C_1D_1$ cube has unit length edges. Find the distance between two circumferences, one of those is inscribed into the $ABCD$ base, and another comes through A, C and B_1 points.

The 20-th competition -- Ulyanovsk, 1986.

form	first day				second day			
8	418	419	420	421	430	431	432	433
9	422	423	424	425	434	433b	435	436
10	426	427	428	429	437	438	439	440

418.
The square polynomial $x^2 + ax + b + 1$ has natural roots. Prove that $(a^2 + b^2)$ is a composite number.

419.
Two equal squares, one with red sides, another with blue ones, give an octagon in intersection. Prove that the sum of red octagon sides lengths is equal to the sum of blue octagon sides lengths.

420.
The point M belongs to the $[AC]$ side of the acute-angle triangle ABC. Two circles are outscribed around ABM and BCM triangles. What M position corresponds to the minimal area of those circles intersection?

421.
Certain king of a certain state wants to build n cities and $n-1$ roads, connecting them to provide a possibility to move from every city to every city. (Each road connects two cities, the roads do not intersect, and don't come through another city.) He wants also, to make the shortests distances between the cities, along the roads, to be $1,2,3,...,n(n-1)/2$ kilometres. Is it possible for
a) $n=6$;
b) $n=1986$?

422.
Prove that it is impossible to draw a convex quadrangle, with one diagonal equal to doubled another, the angle between them 45 degrees, on the coordinate plane, so, that all the vertices' coordinates would be integers.

423.
Prove that the rectangle mxn table can be filled with exact squares so, that the sums in the rows and the sums in the columns will be exact squares also.

424.
Two circumferences, with the distance d between centres, intersect in P and Q points. Two lines are drawn through the A point on the first circumference (Q<>A<>P) and P and Q points. They intersect the second circumference in the B and C points.

a) Prove that the radius of the circle, outscribed around the ABC triangle, equals d.
b) Describe the set of the new circle's centres, if the A point moves along all the first circumference.

425.
Given right hexagon. Each side is divided onto 1000 equal segments. All the points of division are connected with the segments, parallel to sides. Let us paint in turn the triples of unpainted nodes of obtained net, if they are vertices of the unilateral triangle, doesn't matter of what size an orientation. Suppose, we have managed to paint all the vertices except one.
Prove that the unpainted node is not a hexagon vertex.

426.
Find all the natural numbers equal to the square of its divisors number.

427.
Prove that the following inequality helds for all positive $\{a_i\}$.
$1/a_1 + 2/(a_1+a_2) + ... + n/(a_1+...+a_n) < 4(1/a_1 + ... + 1/a_n)$

428.
A line is drawn through the A vertex of ABC triangle with |AB|<>|AC|.
Prove that the line can not contain more than one point M such, that M is not a triangle vertex, and the angles ABM abd ACM are equal.
What lines do not contain such a point M at all?

429.
A cube with edge of length n (n>=3) consists of n^3 unit cubes.
Prove that it is possible to write different n^3 integers on all the

unit cubes to provide the zero sum of all integers in the every row parallel to some edge.

430.
The decimal notation of three natural numbers consists of equal digits: n digits x for a, n digits y for b and $2n$ digits z for c.
For every $n > 1$ find all the possible triples of digits x,y,z such, that $a^2 + b = c$.

431.
Given two points inside a convex dodecagon (twelve sides) situated 10cm far from each other.
Prove that the difference between the sum of distances, from the point to all the vertices, is less than 1m for those points.

432.
Given 30 equal cups with milk. An elf tries to make the amount of milk equal in all the cups. He takes a pair of cups and aligns the milk level in two cups. Can there be such an initial distribution of milk in the cups, that the elf will not be able to achieve his goal in a finite number of operations?

433.
Find the relation of the black part length and the white part length for the main diagonal of the
a) 100x99 chess-board;
b) 101x99 chess-board.

434.
Given right n-angle $A_1A_2...A_n$. Prove that if
a) n is even number, than for the arbitrary point M in the plane, it is possible to choose signs in an expression
   +-[MA_1[+-[MA_2[+- ... +-[MA_n[
   to make it equal to the zero vector (here [MA[denotes vector).
b) n is odd, than the abovementioned expression equals to the zero vector for the finite set of M points only.

435.
All the fields of a square nxn ($n>2$) table are filled with +1 or -1 according to the rules:
At the beginning -1 are put in all the boundary fields.
The number put in the field in turn (the field is chosen arbitraryly) equals to the product of the closest, from the different sides, numbers in its row or in its column.
a) What is the minimal
b) What is the maximal

possible number of +1 in the obtained table?

436.
Prove that for every natural n the following inequality is valid:
$|\sin 1| + |\sin 2| + |\sin(3n-1)| + |\sin(3n)| > 8n/5$.

437.
Prove that the sum of all numbers representable as $1/(mn)$, where
m,n -- natural numbers, $1 \le m < n \le 1986$; is not an integer.

438.
A triangle and a square are outscribed around the unit circle.
Prove that the intersection area is more than 3.4.
Is it possible to assert that it is more than 3.5?

439.
Let us call a polynomial "admissible" if all it's coefficients are 0,
1, 2 or 3. For given n find the number of all the admissible
polynomials P such, that $P(2) = n$.

440.
Consider all the tetrahedrons AXBY, outscribed around the sphere. Let
A and B points be fixed. Prove that the sum of angles in the non-plane
quadrangle AXBY doesn't depend on X and Y points.

The 21-st competition -- Frunze, 1987.

form	first day				second day			
8	441	442	443	444	452	453	454	455
9	445	446a	447	448	456	457	458	459
10	449	450	451	446b	455	460	461	462

441.
Ten sportsmen have taken part in a table-tennis tournament (each pair
has met once only, no draws). Let x_i be the number of i-th player
victories, y_i -- losses.
Prove that ${x_1}^2 + \dots + {x_{10}}^2 = {y_1}^2 + \dots + {y_{10}}^2$

442.
It is known that, having 6 weighs, it is possible to balance the scales
with loads, which weights are successing natural numbers from 1 to 63.
Find all such sets of weighs.

443.
Given right heptagon (7-angle) $A_1 \dots A_7$.

Prove that $1/|A_1A_5| + 1/|A_1A_3| = 1/|A_1A_7|$.

444.
The "Sea battle" game.
a) You are trying to find the 4-field ship -- a rectangle 1x4, situated on the 7x7 playing board. You are allowed to ask a question, whether it occupies the particular field or not. How many questions is it necessary to ask to find that ship surely?
b) The same question, but the ship is a connected (i.e. its fields have common sides) set of 4 fields.

445.
Prove that $(1^{1987} + 2^{1987} + ... + n^{1987})$ is dividable by $(n+2)$.

446.
```
+--+   a) How many such a figures can You put on a 8x8 board without
|  |      intersections?
+--+--+   b) An arbitrary field is cut out of a 1987x1987 board.
|  |  |      Prove that the reminding part of the board can be always
+--+--+      cut on such parts.
```

447.
Three lines are drawn parallel to the sides of the triangles in the opposite to the vertex, not belonging to the side, part of the plane. The distance from each side to the corresponding line equals the length of the side. Prove that six intersection points of those lines with the continuations of the sides are situated on one circumference.

448.
Given two closed broken lines in the plane with odd numbers of edges. All the lines, containing those edges are different, and not a triple of them intersects in one point.
Prove that it is possible to chose one edge from each line such, that the chosen edges will be the opposite sides of a convex quadrangle.

449.
Find a set of five different relatively prime natural numbers such, that the sum of an arbitrary subset is a composite number.

450.
Given a convex pentagon. The angles ABC and ADE are equal. The angles AEC and ADB are equal too.
Prove that the angles BAC and DAE are equal also.

451.
Prove such a, that all the numbers cos a, cos 2a, cos 4a, ... ,

$\cos((2^n)a)$ are negative.

452.
The positive numbers a,b,c,A,B,C satisfy a condition
$a + A = b + B = c + C = k$.
Prove that $aB + bC + cA \le k^2$.

453.
Each field of the 1987x1987 board is filled with numbers, which absolute value is not greater than one. The sum of all the numbers in every 2x2 square equals 0.
Prove that the sum of all the numbers is not greater than 1987.

454.
The B vertex of the ABC angle lies out the circle, and the [BA) and [BC) beams intersect it. The K point belongs to the intersection of the [BA) beam and the circumference. The KP chord is orthogonal to the angle ABC bisector. The (KP) line intersects the BC beam in the M point. Prove that the [PM] segment is twice as long as the distance from the circle centre to the angle ABC bisector.

455.
Two players are writting in turn natural numbers not exceeding p.
The rules forbid to write the divisors of the numbers already having been written. Those who cannot make his move looses.
a) Who, and how, can win if $p=10$?
b) Who wins if $p=1000$?

456.
Every evening uncle Chernomor (see Pushkin’s tales) appoints either 9 or 10 of his 33 "knights" /* let’s call them so */ in the "night guard" /* let’s call it so */. When it can happen, for the first time, that every knight has been on duty the same number of times?

457.
Some points with the integer coordinates are marked on the coordinate plane. Given a set of nonzero vectors. It is known, that if You apply the beginnings of those vectors to the arbitrary marked point, than there will be more marked ends of the vectors, than not marked.
Prove that there is infinite number of marked points.

458.
The convex n-angle ($n \ge 5$) is cut along all its diagonals.
Prove that there are at least a pair of parts with the different areas.

459.

The T_0 set consists of all the numbers, representable as $(2^k)!$, $k = 0, 1, 2, \ldots, n, \ldots$ The T_m set is obtained from T_{m-1} by adding all the finite sums of different numbers, that belong to T_{m-1}. Prove that there is a natural number, that doesn't belong to T_{1987}.

460.
The plot of the $y=f(x)$ function, being rotated by the (right) angle around the $(0,0)$ point is not changed.
a) Prove that the equation $f(x)=x$ has the unique solution.
b) Give an example of such a function.

461.
All the faces of a convex polyhedron are the triangles.
Prove that it is possible to paint all its edges in red and blue colour in such a way, that it is possible to move from the arbitrary vertex to every vertex along the blue edges only and along the red edges only.

462.
Prove that for every natural n helds inequality
$(2n + 1)^n >= (2n)^n + (2n - 1)^n$.


```
 ____/|
 \ o.O|                 Vladimir A. Pertsel
  =(_)=         E-mail:  voldemar@wisdom.weizmann.ac.il
    U      Disclaimer:  This posting represents the poster's views only.
```