Topics in variational methods
(Chapter 5 of the text and some extra..)

Euler-Lagrange equation (5.1)

Different forms of Euler equation
(several dependent variables, higher derivatives, integral constains).
First integrals.
Examples of variational problems:
- Brachistochrone.
- Equilibrium of a chain.
- Minimal surface.

Null-Lagrangians and natural boundary conditions.
Sturm-Luiville problem.

Non-existence of a solution and regularization (example).

Extremals of variational problems:
How to distinguish min from max?
(Weierstrass test and relaxation.)

Multiple integrals:

Euler-Lagrange equations for multiple integrals
Variation of boundary integrals and
variety of natural boundary conditions.
Null-Lagrangians (an example).
Inequalities: Legandre condition.

Lagrange mechanics (5.2)

Hamilton principle. Example.
Link with theory of relativity.
Invariants of mechanical systems (an example).

Examples of variational principles:
- Vibrating string (membrane)
- Conductivity.
- Elasticity.

Numerical methods (5.3):
Ritz and Galerkin methods
Comparing variational methods with solution found
by separation of variables (an example of Dirichlet problem).

Eugenvalue problem (5.4):
minimal principle for the first eigenvalue.
min-max principle for eigenvalues
Examples of eigenvalue problems.
Ritz method.

Several posed but unsolved questions:

Definition of limits of functional sequences.
Weak solutions as limit of Galerkin procedure.
Regularization of ill-posed problems.
Structure of a differential operator (eigenvalues and eigenfunctions).