What is Applied mathematics?

Definition 0.0.1 *Applied mathematics problems are mathematical problems which importance is self-evident.*

Applied math is a group of methods aimed for solution of problems in sciences, engineering, economics, or medicine. These methods are originated by Newton, Euler, Lagrange, Gauss and other giants. Modern areas of applied math include Mathematical physics, Mathematical biology, Control theory, Aero-space engineering, optimal planning, math finance. There is a fuzzy boundary between applied math and engineering and at the other side, between applied and pure mathematics. Applied math discovers new problems which could become subjects of pure math (like geodesics), or develop to become a new engineering discipline (like elasticity theory).

Study of applied math requires expertise in many areas of mathematics and science, physical intuition and common sense, and collaboration skills. Applied math allows for many approaches to the problem, a choice of objectives, and variety of methods.

Definition 0.0.2 *Success in applied math is a new elegant model that catches the phenomenon and novel clever methods to solve it.*

Approaches of applied math

In the majority of applied problems, one need to build mathematical model of a phenomenon, solve this model, and develop the recommendations for improvement of the performance.

Math modeling Typically we deal with large systems (thousands of variables, equations and inequalities). A clever model separates the main phenomenon from the noise and allows for analytic treatment of the problem, followed by extensive numerical development. There may be several models suggested for the same phenomena with various level of details: No one need a one-to-one geographical map. The art of simplification is to make the problem solvable but not trivial. All models have ranges of applicability (Take for example the ideal gas model, compare with the black hole model).

Method of solutions The large variety of applied math problem requires a variety of methods. However, there are several commonly used methods in the applied math workshop. These methods include linear algebra, differential and integral equations, approximation theory and asymptotics, variational principles, and numerics. These will be intensively discussed in the course. The same physical problem can be approached differently, using either statistics, of differential equations, or variational methods, or a combination of them. Typically, there is no single "correct" approach to a complicated problem, each approach highlights different sides of it. Examples: Weather forecast; conflict situation: Worst case scenario or average outcome.

Numerical models allow for solution of classical problems and address novel classes of problems (like math finance, math genetics, weather prediction). They also call for new theoretical approaches: Development of the theory of free boundary problems was caused by the possibility of numerical solution the problem in an arbitrary domain.

Improvement and recommendation When people develop a model of a process, they usually want to improve it. A number of theories answer the question: How to maximize an objective. These are Optimization, variational methods, control theory, game theory. Optimization is the ultimate objective of study of an engineering problem. Sometimes the improvement is achieved by varying the parameters, but generally it is a serious math problem that will be discussed in the class.