m-3150-001 Midterm exam 1

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Your name, ...

Solve any three problems:

1. Find the period of the following functions:

$$a. \qquad f_1(x) = \sin(2x) + \cos(4x) \tag{1}$$
$$b. \qquad f_3(x) = \sin(5x) - 2\cos(3x) \tag{2}$$
$$c. \qquad f_2(x) = \cos(3\pi x) \tag{3}$$

2. a. Find the Fourier series of the function $f(x) = e^x$, $-\pi < x < \pi$.
   b. Differentiate the series and show that $f'(x) = f(x)$.

3. The function 4-periodic is given by formula

$$f(x) = \begin{cases} 3 & \text{if } 0 < x \le 1 \\ -1 & \text{if } 1 < x \le 2 \\ 0 & \text{if } 2 < x \le 4. \end{cases}$$

a. Find the Fourier series.

b. Skatch the graph of the function and of its antiderivative, assuming that integral of the antiderivative over the interval [0, 4] is zero. Write the Fourier series for the antiderivative.

4. Find the even and odd half-range expansions of $f(x) = x^2$, $0 < x < 1$.

   Skatch the graphs of these expansions in the interval $-1 < x < 1$.