HW1

1. Derive the Euler equation. Solve

$$\min_{u(x),b} \int_0^b \left((u')^2 + 2x\,u\right) dx \quad u(0) = 0, \quad u(b) = 0,$$

2. Derive the Euler equation and boundary conditions. Solve. Find the first integral

$$\min_{u(x)} \left[\int_0^1 \left((u')^2 - \omega^2 u^2\right) dx + 2u(1),\right] \quad u(0) = 0,$$

3. Derive the Euler equation and boundary conditions. Solve. Explain the dependence of the solution on a.

$$\min_{u(x)} \int_0^b \left((u')^2 + a\,u\,u' + u\right) dx \quad u(0) = 0.$$

4. Derive the Euler equation and boundary conditions. Solve, using Maple

$$\min_{u(x)} \int_0^b \left((u'')^2 - a\,(u')^2 + b\,u\right) dx \quad u(0) = 0, \quad u'(0) = 0$$