This exam consists of n problems and 1 bonus problem, and will last 60 minutes for some integer n. Answer the questions in the spaces provided. You may use a scientific or graphing calculator. You may not use any books, notes, cell phones etc. You must show all of your work to receive credit. Good luck!

1. Let $y = (x^2 + 2x - 5)(x^3 - 1)$. Compute $D_x y$.

Solution: $D_x y = 5x^4 + 8x^3 - 15x^2 - 2x - 2$

2. Let $g(x) = \frac{x+1}{2-x^2}$. Compute $g'(x)$.

Solution: $g'(x) = \frac{x^2+2x+2}{(2-x^2)^2}$

3. Let $f(t) = \frac{t^3-1}{t^2+3t-2}$. Find $f'(t)$.

Solution: $f'(t) = \frac{t^4+6t^3-6t^2+2t+3}{(t^2+3t-2)^2}$

4. Find an equation of the tangent line to $f(x) = \sqrt{x} + 5$ at the point $(9, 8)$.

Solution: $x - 6y = -39$

5. If $f(x) = \frac{1}{x} + \cos x$, find $f'(x)$.

Solution: $f'(x) = -\frac{1}{x^2} - \sin x$

6. Let $g(x) = 3\tan(x) - 2\sec(x) + 3x^2$. Find $g'(x)$.

Solution: $g'(x) = 3\sec^2(x) - 2\sec(x)\tan(x) + 6x$

7. Let $y = (x^2 - x + 5)^4$. Find $\frac{dy}{dx}$.

Solution: $\frac{dy}{dx} = 4(2x-1)(x^2-x+5)^3$

8. Let $w = \csc^2(t)$. Find $\frac{dw}{dt}$.

Solution: $\frac{dw}{dt} = -2\csc^2(t)\cot(t)$

9. Let $y = \cos(x^3)$. Find $\frac{d^2y}{dx^2}$.

Solution: $\frac{dy}{dx} = -3x^2\sin(x^3)$
$\frac{d^2y}{dx^2} = -9x^4\cos(x^3) - 6x\sin(x^3)$

10. At time t, the position of a particle moving along a line is given by $s = 3t^8 - 2t^5$. Find the velocity and acceleration as functions of time.

Solution: Velocity $= \frac{ds}{dt} = 24t^7 - 10t^4$
Acceleration $= \frac{d^2s}{dt^2} = 168t^6 - 40t^3$

11. Use implicit differentiation to find $\frac{dy}{dx}$ if $x^2y + xy^2 = 5y$.

Solution: $\frac{dy}{dx} = \frac{2xy+y^2}{5-x^2-2xy}$

12. Use implicit differentiation to find $\frac{du}{dw}$ if $w^3 - 2 + wu + u^3 + 24 = 0$

Solution: $\frac{du}{dw} = \frac{-3w^2-u}{w+3u^2}$

13. Use implicit differentiation to find $\frac{dy}{dx}$ if $\cos(xy) = x$.

Solution: $\frac{dy}{dx} = -\frac{y\sin(xy)+1}{x\sin(xy)}$

14. A spherical balloon is inflated at a rate of 4 ft^3/min. How fast is the radius increasing when the radius is 2 ft? (The volume of a sphere of radius r is $V = \frac{4}{3}\pi r^3$.)

Solution: $\frac{1}{4\pi}$ ft/min

15. An extension ladder leaning against a wall is collapsing at a rate of 2 ft/sec while the foot of the ladder remains a constant 5 ft from the wall. How fast is the ladder moving down the wall when the ladder is 13 ft long?

Solution: $\frac{13}{6}$ ft/sec

16. Let $f(x) = \frac{1}{x^2}$. Find a general expression for dy. Evaluate dy if $x = -2$ and $dx = 0.5$.

Solution: $dy = -\frac{2}{x^3}dx$
$dy = 0.125$

17. Approximate $\sqrt{98}$ using differentials. Round your answer to EXACTLY 5 decimal places.

Solution: 9.90000

18. Let $y = 3x^2 - 5$. Compute dy and Δy if x changes from 2 to 2.1.

Solution: $dx = 0.1$
$dy = 1.2$
$\Delta y = 1.23$

19. The radius of a circle is measured as 17 cm with an error of $\pm 0.4cm$. Estimate the error in calculating the circumference of the circle.

Solution: $\pm 0.8\pi$ cm

20. Determine the maximum and minimum values of the function $f(x) = x^3 - 3x^2 + 2$ on the interval $[-1, 1]$.

Solution: maximum: 2
minimum -2

21. Determine the maximum and minimum values of the function $f(x) = \frac{x}{x^2+1}$ on the interval $[-1, 0]$.

Solution: maximum: $= 0$
minimum: $-\frac{1}{2}$

22. Determine the intervals where $f(x) = \frac{1}{3}x^3 + x^2 - 3x$ is increasing.

Solution: $(-\infty, -3), (1, \infty)$.

23. Determine the intervals where $f(x) = \frac{8x}{x^2+4}$ is decreasing.

Solution: $(-\infty, -2], [2, \infty)$

24. Let $f(x) = \frac{x}{x^2+1}$. Determine the intervals where $f(x)$ is concave up, the intervals where $f(x)$ is concave down and find all points of inflection.

Solution: Concave up: $(-\sqrt{3}, 0), (\sqrt{3}, \infty)$
Concave down: $(-\infty, -\sqrt{3}), (0, \sqrt{3})$
Points of inflection: $(0, 0), (-\sqrt{3}, -\frac{\sqrt{3}}{4}), (\sqrt{3}, \frac{\sqrt{3}}{4})$.

25. Let $f(x) = x^3 - 6x^2 + 9x + 3$. Determine the intervals where $f(x)$ is increasing, intervals where $f(x)$ is decreasing, intervals where $f(x)$ is concave up, intervals where $f(x)$ is concave down, and find all points of inflection.

Solution: increasing: $(-\infty, 1], [3, \infty)$
decreasing: $[1, 3]$
concave up: $(2, \infty)$
concave down:$(-\infty, 2)$
points of inflection: $(2, 5)$

26. Find all points which give a local maximum and all points which give a local minimum value for $f(x) = x^3 - 6x^2 + 9x + 3$.

Solution: local maximum at $x = 1$, local minimum at $x = 3$.

27. Find all points which give a local maximum and all points which give a local minimum for the function $f(x) = \frac{8x}{x^2+4}$.

Solution: local maximum at $x = 2$
local minimum at $x = -2$

28. A rectangular field is to be enclosed by a fence and divided into 3 lots by fences parallel to one of its sides. Find the dimensions of the largest field that can be enclosed with a total of 800 meters of fencing.

Solution: 100 meters by 200 meters

29. An open box is formed from a square sheet of cardboard by cutting equal squares from each corner and folding up the edges. If the dimensions of the cardboard are 18cm by 18cm, what should be the dimensions of the box so as to maximize the volume? What is the maximum volume?

Solution: Dimensions: 3cm by 12cm by 12cm.
The maximum volume is 432 cm^3.

30. A motorist is in a desert in a jeep. The closes point on a straight road is town A and it is $4\sqrt{2}$ miles from the motorist (he is not on the road). He wishes to reach town B, which is 10 miles down the road from town A. If he can drive 15 miles per hour on the desert and 45 miles per hour on the road, where should he intersect the road to minimize his total driving time? (Give your answer as either a distance from town A or a distance from town B. Be sure to specify which!)

Solution: 2 mi from A, or 8 mi from B