Homework 2

1. Show that multiplication is well defined in $\mathbb{Z}/m$, i.e. if $x \equiv x'$ and $y \equiv y'$ then $xy \equiv x'y'$.

2. Let G be a group and X a subset of G. Let $\langle X \rangle$ be the intersection of all subgroups of G that contain X. Show that $\langle X \rangle$ is in fact a subgroup, called *subgroup generated by* X. If $G = GL_n(\mathbb{R})$ and X is the set of all elementary matrices, what subgroup is $\langle X \rangle$?

3. Artin 2.2.4.

4. Artin 2.4.7.

5. Artin 2.4.9.

6. Artin 2.9.7.

7. (Bonus Problem) Perhaps you wondered what the order of a product xy might be if the orders of x and y are known to be $a > 1$ and $b > 1$ respectively. The answer is: anything! Prove this for $a = b = 2$, i.e. construct a group G and elements $x, y \in G$ of order 2 so that xy has order n for a given integer $n \geq 1$.