Merry Carney

Lesson Plan

The two classes I want to use this with is Secondary Math 3 Essentials and College Prep. The Essentials class smaller class of students that can't pass in a regular Sec 3 class. There are usually a number of resource students mixed with kids that either have never felt successful in math or for some reason (usually drugs or truancy or sickness) missed out on most of some other year of high school math. So I see these students every school day for 80 minutes. (That is a lot of math for kids who do not really "like" math.) The college prep class is for seniors who do not want to take anything beyond Sec 3 but do plan to go to college and want to keep up on their math skills. This is also used as a way for kids who have failed a quarter of Sec 3 to make up that credit. I find most of these students need to learn everything again from the very basics and with lots of extra practice. This is even true for the "smart" but lazy students that drop down from Calculus or AP stats. So that is the audience.

Lesson: I want to share a short lesson on complex numbers. One that I have used but then add vectors to it. This lesson will be taught before the unit circle, usually showing up while solving polynomials. My main focus will be in teaching the unit circle.

Common Core Standards:

N.CN.2 Use the relation i2 = -1 and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.
F.TF.1 Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
F.TF.2 Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
FTF.3 Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and π /6, and use the unit circle to express the values of sine, cosine, and tangent for $\pi - x$, $\pi + x$, and 2 $\pi - x$ in terms of their values for x, where x is any real number.
Complex Numbers

Prior knowledge: None of these students would have learned about vectors unless they have taken physics or happened to be in honors math at some point. They will have already seen complex numbers and have learned the basic trigonometric ratios for finding missing sides and angles as well as having been introduced to radian measures.

Complex Numbers using Vectors and Rotations:

The students will have already seen and worked with imaginary and complex numbers. So this will be a reminder of what they are. However, when I taught this last year, none of my students had seen imaginary numbers as rotations around the origin. I want to take what I did last year and add the idea of $i \text{ and } a + bi$ as vectors. The following table was found at the first reference given below. There is an excellent video to go with it but I may use next time but last year simply used what he said and taught it myself.

Fun Fact	Negative Numbers (-x)	Complex Numbers (a + bi)
Invented to answer	"What is 3 – 4?"	"What is sqrt(-1)?"
Strange because...	*How can you have less than nothing?*	*How can you take the square root of less than nothing?*
Intuitive meaning	"Opposite"	"Rotation"
Considered absurd until	1700s	Today ☺
Multiplication cycle [& general pattern]	`1, -1, 1, -1...` `X, -X, X, -X...`	`1, i, -1, -i...` `X, Y, -X, -Y...`
Use in coordinates	Go backwards from origin	Rotate around origin
Measure size with	Absolute value $\sqrt{(-x)^2}$	Pythagorean Theorem: $\sqrt{a^2 + b^2}$

https://betterexplained.com/articles/a-visual-intuitive-guide-to-imaginary-numbers/

https://www.khanacademy.org/math/linear-algebra/vectors-and-spaces/vectors/v/vector-introduction-linear-algebra

Lesson:

- Starter to create vectors before knowing what they are and to remember how to find the length of a line segment using Pythagorean Theorem.
- Introduce vectors with the notes.
- Show the results of rotating i to generate $i^2, i^3, i^4, \ldots$
- Show complex numbers on the coordinate plane as vectors and see how adding the complex numbers and the picture of adding the vectors coincide.

Lesson on the Unit Circles:

I want to use rotations and reflections as a way of helping the students understand and remember the unit circle. This will be used after already working with right triangles, finding sides and angle using trig ratios as well as working with special right triangles. I will then use our rotation and reflection matrices to generate the points. I cannot assume that any of the students will have ever worked with matrices before. We do not have time to get into matrices but maybe this will wet a bit of an appetite and will let them connect the points being (cos a, sin a) with the actual transformations. I will simply present these matrices and work with them to see how they work.) Radian measure will be after this lesson.

- Starter to review special right triangles.
- "Test of your Memory" to review reflections and rotations.
- Notes on the unit circle with time taken to rotate at 45°, 30°, and 60°. At this point I will prove why the corresponding point are (cos a, sin a)
- Show this before moving on http://www.mathsisfun.com/geometry/unit-circle.html
- Give a blank unit circle with appropriate lines and have students fill in themselves as they begin to memorize. Also have them list and discuss all the similarities, rotations, and reflections.
- As an extension introduce our rotation and reflection matrices and show them how they can be used to find the points on the "Test your Memory" worksheet and how to get the points on the unit circle.
- Extension: Find points one the unit circle using the rotation and reflection matrices. Show this as vectors as the columns of the matrix and explain with a picture.

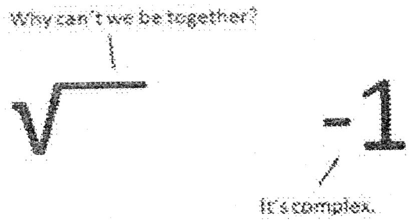
Rotations = $\begin{bmatrix} \cos a & -\sin a \\ \sin a & \cos a \end{bmatrix}$ Reflections = $\begin{bmatrix} \cos 2a & \sin 2a \\ \sin 2a & -\cos 2a \end{bmatrix}$

http://www.cpalms.org/Public/PreviewResourceLesson/Preview/46782

http://www.mathsisfun.com/geometry/unit-circle.html

http://www.mathsisfun.com/algebra/trig-interactive-unit-circle.html

http://www.onlinemathlearning.com/unit-circle-symmetry-periodicity-hsf-tf4.html



You can't even **imagine** the fun we will have today!

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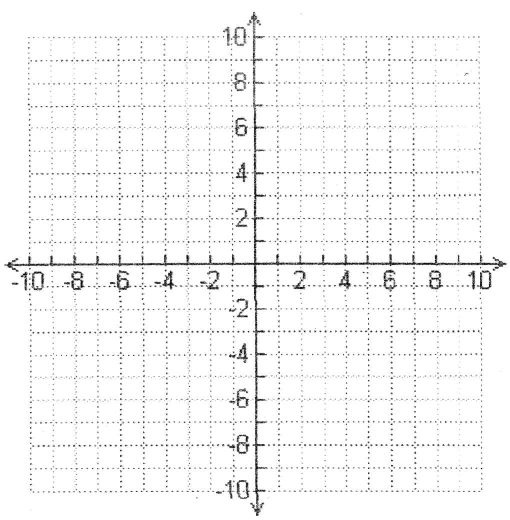
Starter:

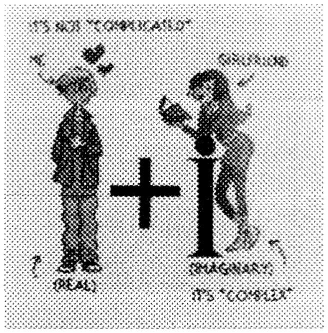
1) Graph the points (3,4) and (-4,3) on the graph.

2) Draw a line from (0,0) to each point. **(You should now have two line segments!)**

3) Find the length of each of the lines you have just drawn. Write this number on each line.

4) Share your answer with your neighbor.





A look at *i* and Complex Numbers using

Vectors and Rotations

Vectors: This will be a minimal introduction to vectors!

A vector has a magnitude and a direction. Without a directions it is called a scaler.

Example: Carney lives 1.5 miles from Skyline. I can draw a line from Skyline to my house. That 1.5 miles is a scaler. If I added that I live 1.5 miles to the north of Skyline now my line has a magnitude (1.5 miles) and a direction (north). Vectors are useful in many areas of math and science but for our purposes we will mainly be looking at a positional vector that begins at (0,0) and ends at a point on the coordinate plane as in our starter. Let's draw a picture and talk about the following:

Vector notation:

Unit Vector:

Zero Vector:

A vector times by a number:

- Vector:

|V|:

Adding vectors:

Complex Numbers $a + bi$ and $i = \sqrt{-1}$ *and* $i^2 = -1$

Fun Fact	Negative Numbers (-x)	Complex Numbers (a + bi)
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Use in coordinates	Go backwards from origin	Rotate around origin
Measure size with	Absolute value $\sqrt{(-x)^2}$	Pythagorean Theorem: $\sqrt{a^2 + b^2}$

Take a look on the coordinate plane:

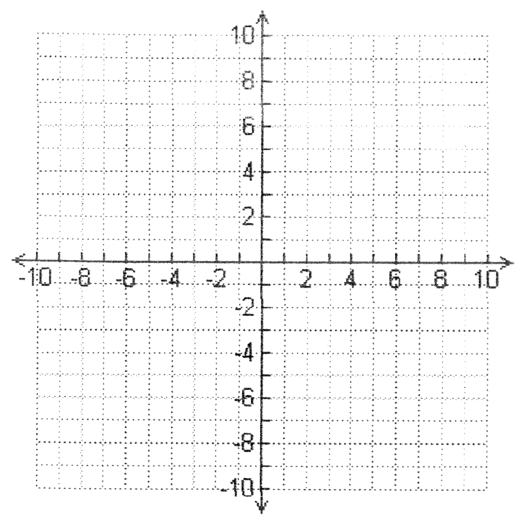
1) Where is i? i^2?

2) Find $i^3 =$ ____

$i^4 =$ ____

$i^5 =$ _____

$i^6 =$ ____

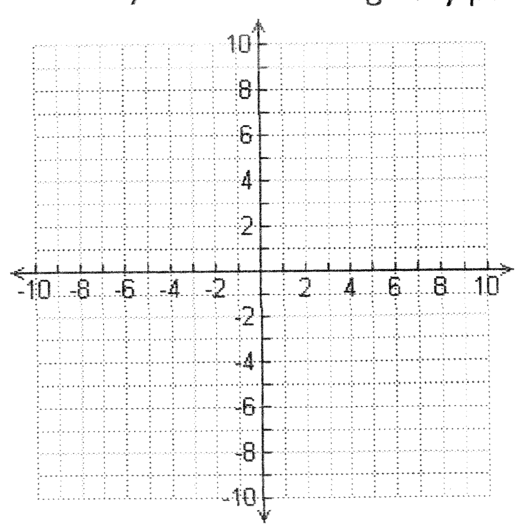


To graph complex numbers we use the complex coordinate grid. This simply means that the x-axis is the real part of the number (a) and the y-axis is the imaginary part (bi).

Graph the following numbers with me:

$2 + 3i, 4 + 2i, 0 - 5i, 2 + 0i$

$(2 + 3i) + (4 + 2i)$



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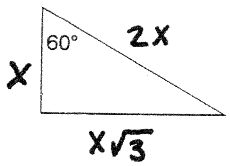
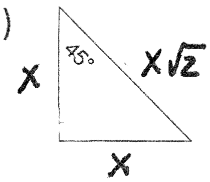
Show (prove) that each of these are right triangles.

1)



2)



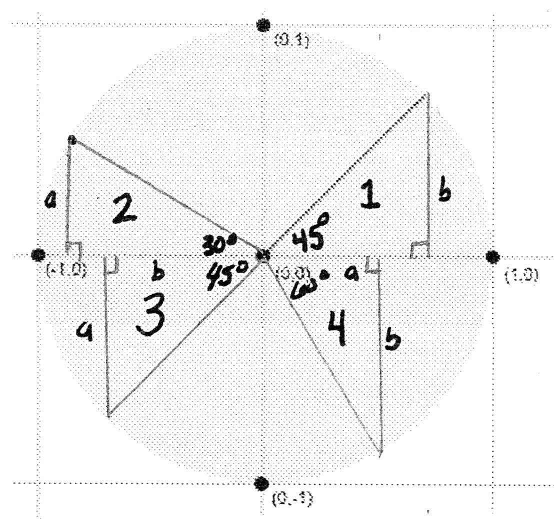
For each of the following the hypotenuse is equal to one. Using that find the length of the other two sides. Use the Pythagorean Theorem to show that you are right. Leave your answers as radicals in simplest form.

3) Triangle 1:

4) Triangle 2:

5) Triangle 3:

6) Triangle 4:



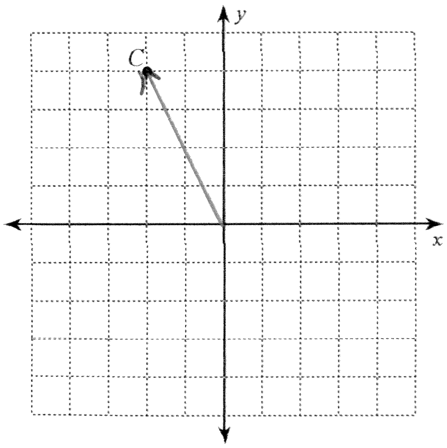
Secondary 3 Name________________________________ ID: 1

Test of your Memory

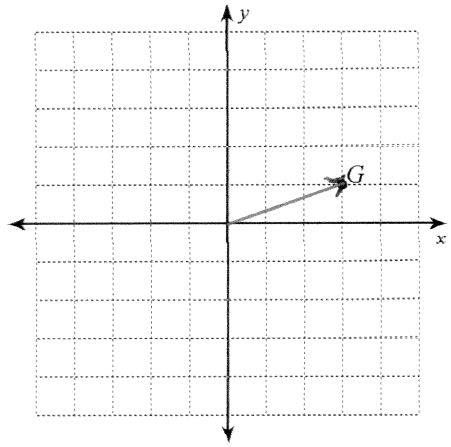
Date________________ Period____

Graph the image of the figure using the transformation given.

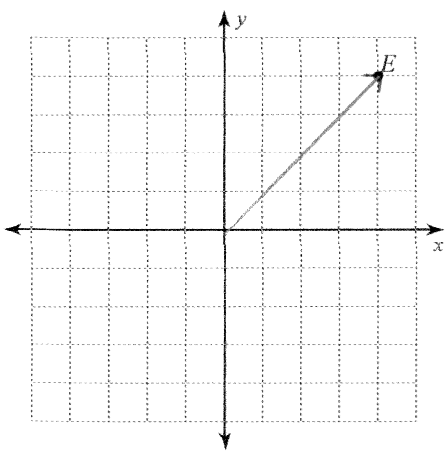
1) reflection across the y-axis



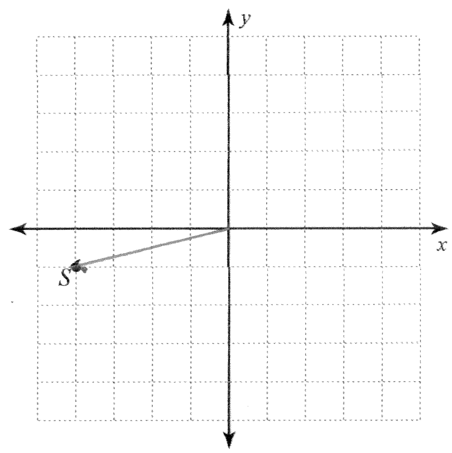
2) reflection across the x-axis



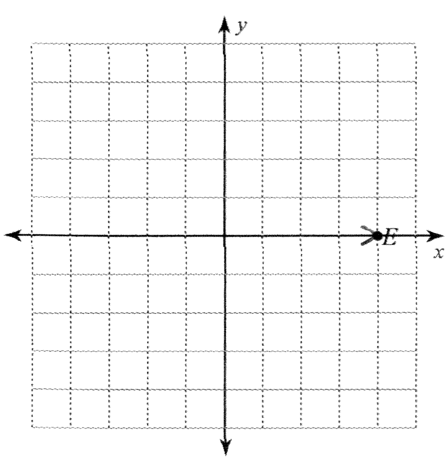
3) rotation 90° counterclockwise about the origin



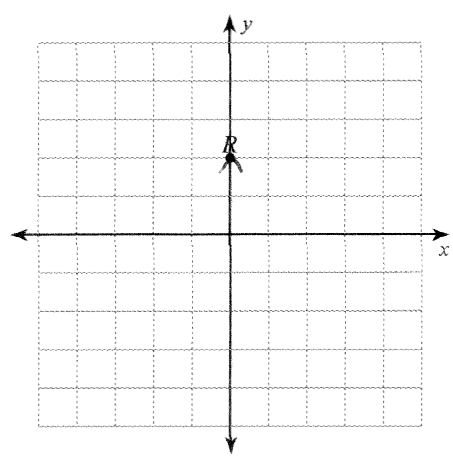
4) rotation 180° about the origin



5) rotation 60° counterclockwise about the origin



6) rotation 45° counterclockwise about the origin

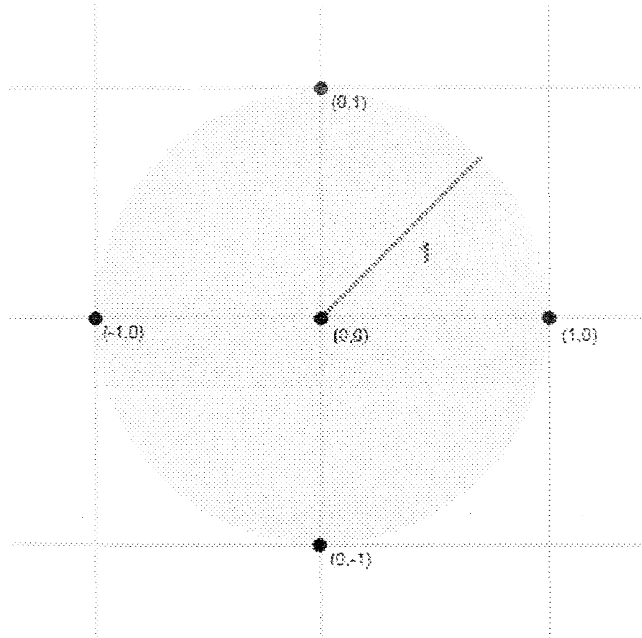


Introduction to the Unit Circle.

Equation is $x^2 + y^2 = 1$

Center is at the origin (0,0)

Radius length of 1 unit



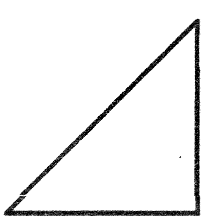
1) To create the rest of the Unit Circle begin by putting in the degrees that go with the points (1,0), (0,1), (-1,0), and (0,-1). (These are our unit vectors.)

2) Next we will rotate the unit vector (1,0) around the circle at 45° until we return to the (1,0).

3) Remembering the 45-45-90 triangles from the starter, we will now put in the coordinates of each corresponding point on the circle for each of our rotations.

I claim that each of these points is (cos 45°, sin 45°). Why is that true?

Proof:



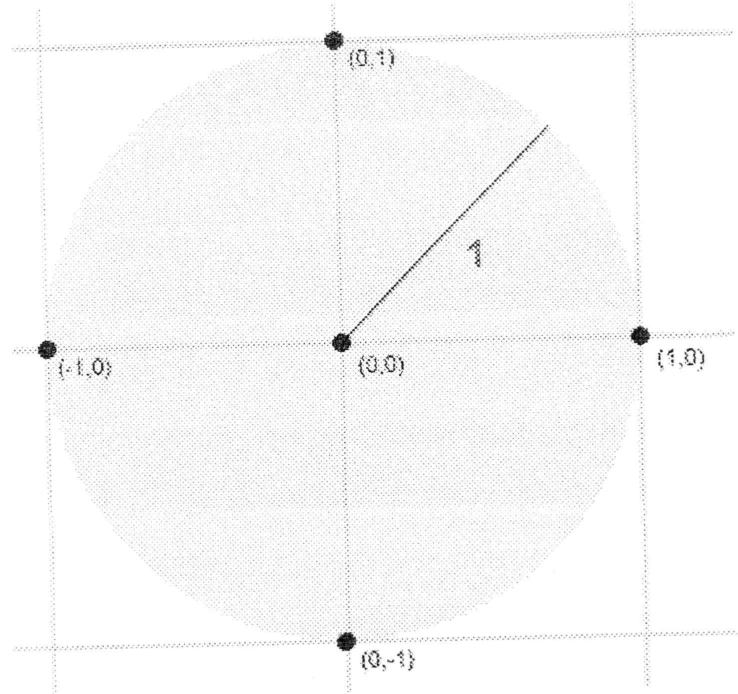
Hypotenuse = 1

Sides = ______

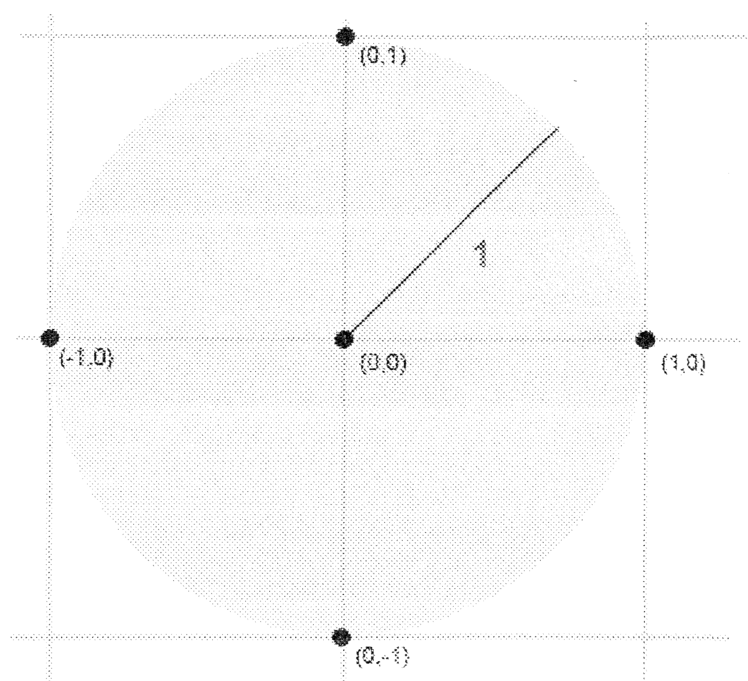
Sin 45 =

Cos 45 =

For this next Unit Circle rotate the unit vector (1,0) around the circle at 60°. When you are done, label the coordinates of each corresponding point on the circle. Convince yourself that this point is (cos 60°, sin 60°).



Finally, for this Unit Circle rotate the unit vector (1,)) around the circle at 30°. When you are done, label the coordinates of each corresponding point on the circle. Convince yourself that this point is (cos 30°, sin 30°).



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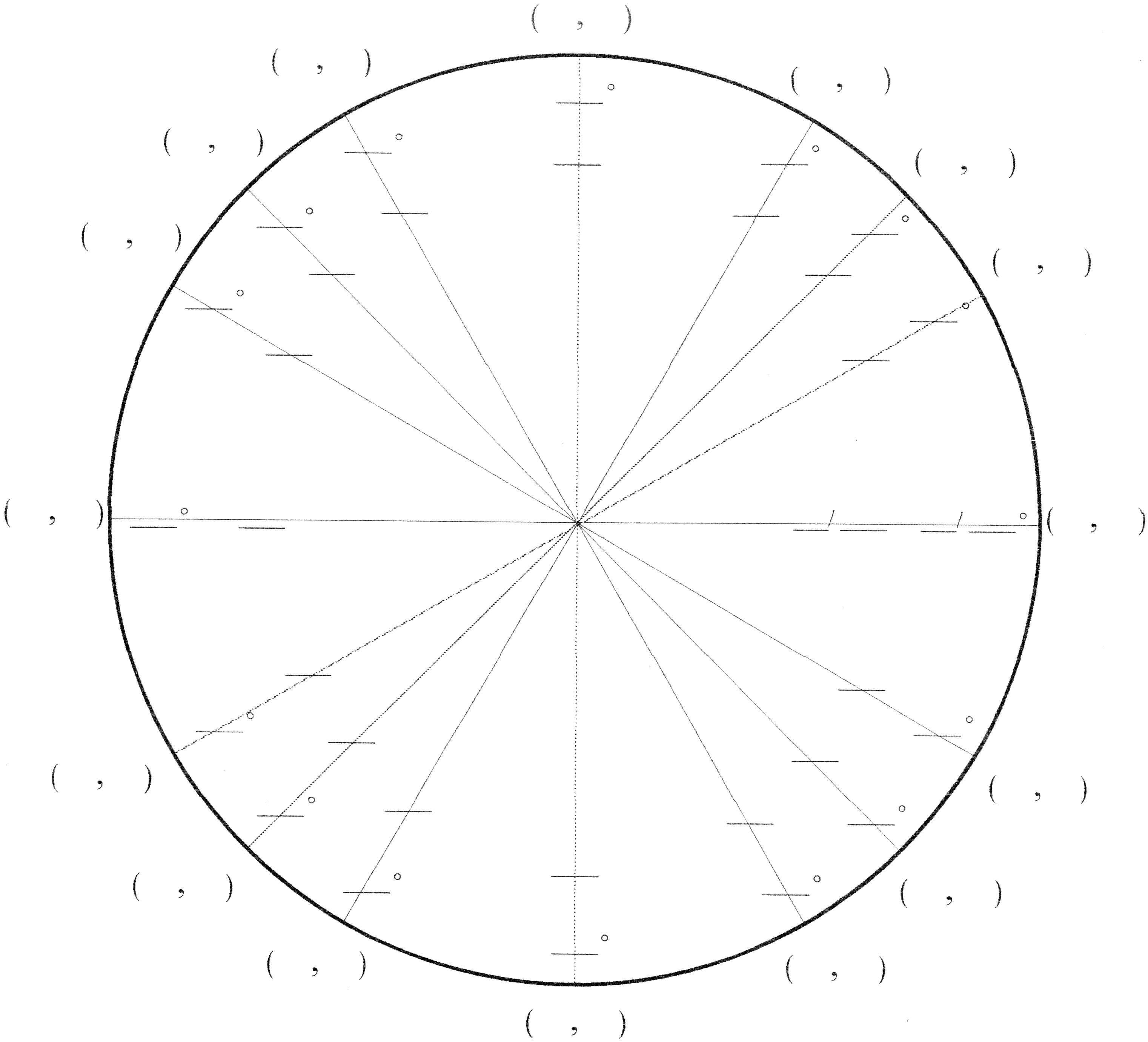


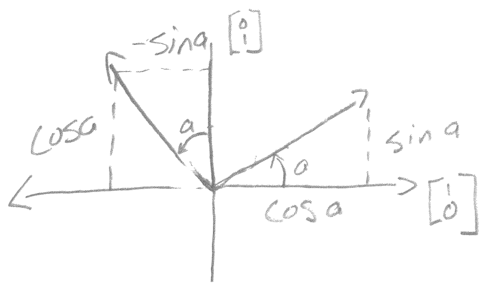
List all the similarities, rotations, and reflections. What can you see that will help you remember?

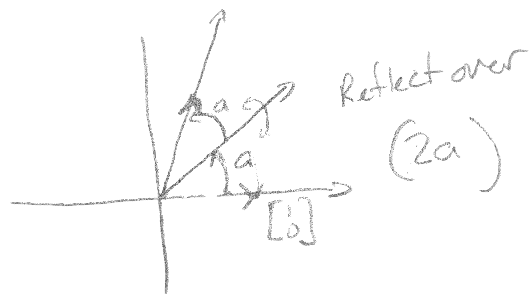
Name______________________________

Something extra cool!

We will use these two matrices to generate some of the points on the unit circle created through rotations and reflections.

Rotations = $\begin{bmatrix} \cos a & -\sin a \\ \sin a & \cos a \end{bmatrix}$ Reflections = $\begin{bmatrix} \cos 2a & \sin 2a \\ \sin 2a & -\cos 2a \end{bmatrix}$





Try a few:

1) 30° $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ reflect to get the point at 150°.

2) 45° $\left(\frac{\sqrt{2}}{2}, \frac{2}{2}\right)$ rotate 15°

3) You name one:

Do two more of your choice.

4)

5)