

**Lesson Plan**

**Name: Jessica Humphrey**

**Date:**

**Time: 88 minutes**

**Grade Level: 11**

**Subject: Secondary Mathematics 3H**

1. **Utah State Core Curriculum Standard(s)**

Standard G.SRT: Apply trigonometry to general triangles.

1. **Lesson Objective(s) / Goal(s)**

Prove the Law of Sines. Understand and apply the Law of Sines to find unknown measurements in right and non-right triangles.

1. **Preparation**
	1. **Teacher**
* Law of Sines MathXL homework created
* Right triangle trigonometry problems for starter (attached)
* Law of Sines problems for starter/practice (attached)
* Law of Sines speed dating cards (not attached – I have them in my classroom)
* Set up the desks for speed dating
	1. **Student**
* Prior knowledge of basic trigonometric ratios (SohCahToa)
* Prior knowledge of basic vectors
1. **Technology Use**

None

1. **Instructional Procedures**

**Starter**

Begin with a starter containing problems where students must use basic trigonometric ratios, sine, cosine, and tangent, to find side lengths of right triangles (see attached). At the end, include one problem with a missing side length of a non-right triangle (Law of Sines problem – see attached). The students should be able to move quickly through the first problems (they may need a reminder of SohCahToa), but should struggle with the last problem.

Give the students time to struggle and encourage them to find another way to solve the last problem. At the right time, bring the class back together to answer the problems. When it comes to the last problem, assuming nobody has the correct answer, ask why this problem was different. The students should be able to point out that the last triangle is not a right triangle and so we cannot use the basic trigonometric ratios. Tell the students that we need to come up with a more complex set of ratios, or a formula if they prefer, to help us solve the last problem. Leave this problem on the far side of the whiteboard where the students can still see it, but it is not the focus.

**Derive the Law of Sines**

Start the lesson with a new triangle made from vectors, still non-right, with vectors $A, B, C$ and corresponding angles $α, β, γ$.

Tell the students we will begin by finding the area of this triangle. Take suggestions and try them on the board, allowing the students plenty of time to think about the problem. Assuming nobody has the correct answer, tell them to first create parallelogram $AB$ with diagonal $A+B$ and then find the area of the parallelogram. They should know the formula for the area of the parallelogram, $A=bh$ where *b* is the base and *h* is the height. Ask the students what the height of the triangle is, they should lead you to drop an altitude through $β$ perpendicular to $B$. Ask the students concerning right-triangle trigonometry (opposite, hypotenuse, adjacent), according to $γ$ what is this altitude? They should answer opposite; label the altitude so. So now we have $A\_{parallelogram}=\left|B\right|⋅\left|opp\right|$. But this doesn’t help us because we don’t know what the magnitude of the opposite is, so we need to incorporate more trigonometry. Ask the students according to $γ$ what is $A$? They should answer hypotenuse. So we can use $\sin(\left(γ\right))\rightarrow A\_{p}=\left|B\right|⋅\left|\frac{opp}{hyp}\right|\rightarrow A\_{p}=\left|B\right|⋅\sin(\left(γ\right))$. But now we’ve changed the equation by dividing by the magnitude of the hypotenuse, so we need to multiply by the magnitude of the hypotenuse which is the same as multiplying by the magnitude of $A\rightarrow A\_{p}=\left|B\right|⋅\sin(\left(γ\right))⋅|A|$. Therefore the area of the triangle is $A\_{triangle}=\frac{1}{2}\left|A\right|\left|B\right|\sin(\left(γ\right))$. Likewise, had we created parallelogram $BC$ or $AC$, $A\_{t}=\frac{1}{2}\left|B\right||C|sin⁡(α)$ and $A\_{t}=\frac{1}{2}\left|A\right||C|sin⁡(β)$ (write these equations on the board next to the first one). If the students are not convinced of this, you may need to go through the process all over again.

Now tell the students that you’re going to manipulate these equations and to just watch/ask questions if they believe you made an algebraic mistake. Multiply each equation by 2, divide by $\left|A\right|\left|B\right|\left|C\right|$, and simply:

$$A\_{t}=\frac{1}{2}\left|A\right|\left|B\right|\sin(\left(γ\right))$$

$$2A\_{t}=\left|A\right|\left|B\right|\sin(\left(γ\right))$$

$$\frac{2A\_{t}}{\left|A\right|\left|B\right||C| }=\frac{\left|A\right|\left|B\right|\sin(\left(γ\right))}{\left|A\right|\left|B\right||C|}$$

$$\frac{2A\_{t}}{\left|A\right|\left|B\right|\left|C\right|}=\frac{\sin(\left(γ\right))}{\left|C\right|}$$

$$A\_{t}=\frac{1}{2}\left|B\right|\left|C\right|\sin(\left(α\right))$$

$$2A\_{t}=\left|B\right|\left|C\right|\sin(\left(α\right))$$

$$\frac{2A\_{t}}{\left|A\right|\left|B\right||C| }=\frac{\left|B\right|\left|C\right|\sin(\left(α\right))}{\left|A\right|\left|B\right||C|}$$

$$\frac{2A\_{t}}{\left|A\right|\left|B\right|\left|C\right|}=\frac{\sin(\left(α\right))}{\left|A\right|}$$

$$A\_{t}=\frac{1}{2}\left|A\right|\left|C\right|\sin(\left(β\right))$$

$$2A\_{t}=\left|A\right|\left|C\right|\sin(\left(β\right))$$

$$\frac{2A\_{t}}{\left|A\right|\left|B\right||C| }=\frac{\left|A\right|\left|C\right|\sin(\left(β\right))}{\left|A\right|\left|B\right||C|}$$

$$\frac{2A\_{t}}{\left|A\right|\left|B\right|\left|C\right|}=\frac{\sin(\left(β\right))}{\left|B\right|}$$

Now since the left sides of these equations are all the same, we can set the right sides equal: $\frac{\sin(\left(α\right))}{\left|A\right|}=\frac{\sin(\left(β\right))}{\left|B\right|}=\frac{\sin(\left(γ\right))}{\left|C\right|}$. Ask the students if this new set of ratios will help us with our original problem. Give them some time to use the new information to solve the original problem – give guidance to those who need it. After sufficient time, bring the class back together and walk them through solving the original problem. Tell the students that this new set of ratios is called the Law of Sines.

**Practice Using the Law of Sines**

Give the students similar problems to practice (see attached). Depending on the needs of the class, you might want to walk through a problem or two more before letting the students do it on their own. Once most students are confident in their ability to solve a problem, pass out the Law of Sines speed dating cards. Play Law of Sines speed dating for the remainder of the class.

Just before the end of class, assign the MathXL homework. Collect the speed dating cards and have the students clean up, including putting class calculators away.

1. **Accommodation(s) for Diverse Learner(s)**
* Speed dating cards are designed such that students will receive a problem suited to their needs (easy, average, or difficult).
1. **Evaluation/Assessment of Student Progress**
	1. **Pre-Assessment**

The starter at the beginning of class will allow me to see how many of the students begin to come up with a solution close to the Law of Sines. This will give me the information I need to gauge how slow or fast I will take the derivation of the Law of Sines. A few practice problems after deriving the Law of Sines will allow me to decide which students need which speed dating cards (easy, average, or difficult).

* 1. **Formative Assessment**

Through the speed dating game, students will understand and apply the Law of Sines to find unknown measurements in right and non-right triangles.

* 1. **Summative Assessment**

At the end of the unit there will be a written test in which students will need to understand and apply the Law of Sines to find unknown measurements in right and non-right triangles.