

Name KEY  
(print)

Total = 100 points

Please show all your work.

1. (20 pts) Forty-three percent of businesses in the U.S. require a doctor's note when an employee takes sick time. You randomly select nine businesses and ask each if it requires a doctor's note when an employee takes sick time.

a) (10 pts) Find the probability that the number who say they require a doctor's note is more than 7.

$$\begin{aligned}
 P(8) + P(9) &= \\
 &= {}_9C_8 (0.43)^8 (0.57)^1 + {}_9C_9 (0.43)^9 (0.57)^0 = \\
 &= 0.005996 + 0.000503 = 0.006499 \approx 0.0065 \\
 &\quad \text{or } \approx 0.006
 \end{aligned}$$

$n=9$   
 $p=0.43$   
 $q=0.57$

Answer 0.0065  
0.006 OK

b) (10 pts) Find the mean and standard deviation.

$$\begin{aligned}
 \bar{x} &= n \cdot p \\
 \bar{x} &= 9 \cdot 0.43 = 3.87
 \end{aligned}$$

$$\begin{aligned}
 s &= \sqrt{n \cdot p \cdot q} \\
 s &= \sqrt{9 \cdot 0.43 \cdot 0.57} \\
 s &= 1.485
 \end{aligned}$$

Mean 3.87 ≈ 3.9Standard deviation 1.49 ≈ 1.5

2. (10 pts) 1,400 raffle tickets are sold at \$2 each for 3 prizes valued at \$800, \$600 and \$300. You buy one ticket. What is the expected value of your gain/loss?

$$E(x) = \text{mean} = \bar{x}$$

$$\begin{aligned}
 \bar{x} &= \sum x \cdot P(x) = -2 \cdot \frac{1397}{1400} + 798 \cdot \frac{1}{1400} + 598 \cdot \frac{1}{1400} + 298 \cdot \frac{1}{1400} = \\
 &= -0.7857 \approx -0.79 \$
 \end{aligned}$$

x	P(x)
-2	$\frac{1397}{1400}$
798	$\frac{1}{1400}$
598	$\frac{1}{1400}$
298	$\frac{1}{1400}$

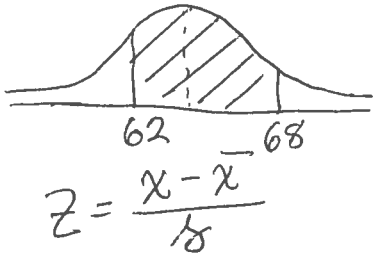
Answer loss of .79 \$  
or -0.79 \$

3. (24 pts) In a survey of women in the U.S. (ages 20-29) the mean height was 64.3 inches with a standard deviation of 2.6 inches. Assume that the heights are normally distributed.

$$\bar{x} = 64.3$$

$$s = 2.6$$

a) (8 pts) Find the probability that the height of a randomly selected woman (age 20-29) is between 62 and 68 inches?



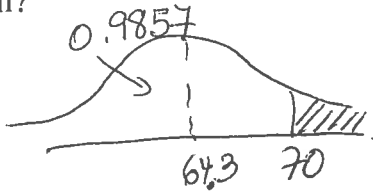
$$z_{62} = \frac{62 - 64.3}{2.6} = -0.88$$

$$z_{68} = \frac{68 - 64.3}{2.6} = 1.42$$

$$P(62 < X < 68) = P(-0.88 < Z < 1.42) = 0.9222 - 0.1894$$

Answer 0.7328

b) (8 pts) If you randomly select 300 women, about how many of them will be over 70 inches tall?



$$P(X > 70) = P(Z > 2.19) = 1 - 0.9857$$

$$z_{70} = \frac{70 - 64.3}{2.6} = 2.19$$

$$= 0.0143$$

$$0.0143 \times 300 = 4.29 \approx 4$$

Answer about 4 women

c) (8 pts) What height represents the third quartile?



75% is below

$$z = 0.67$$

$$0.67 = \frac{x - 64.3}{2.6}$$

$$x = 66.042 \approx 66.04 \text{ in}$$

Answer 66.04 in

4. (16 pts) The annual per capita consumption of ice cream in the U.S. can be approximated by a normal distribution, with a mean of 15.4 lb and a standard deviation of 2.5 lb.

a) (8 pts) Find the annual per capita consumption that represents the 38<sup>th</sup> percentile? (the word quartile should not be found here)

$$\bar{x} = 15.4 \text{ lb}$$

$$s = 2.5 \text{ lb}$$

$$-0.31 = \frac{x - 15.4}{2.5}$$

(multiply both sides by 2.5; then add 15.4 to both sides)

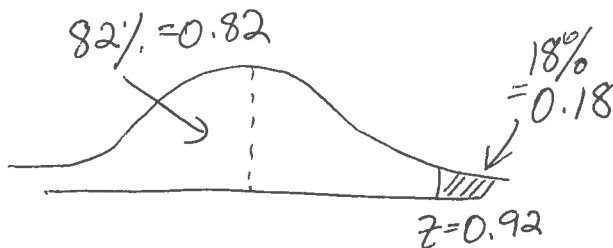
$$x = 14.749 \approx 14.75 \text{ lb}$$

38% is below (using the table)

$$z = -0.31$$

$$\text{Answer } \frac{14.625 \text{ lb}}{\approx 14.63 \text{ lb}}$$

b) (8 pts) Find the smallest annual per capita consumption of ice cream that can be in the top 18% of consumptions?



$z = 0.92$  (found from the table)

$$0.92 = \frac{x - 15.4}{2.5}$$

$$x = 17.7 \text{ lb}$$

Answer 17.7 lb

5. (20 pts) "How many cups of coffee do you drink?" was asked on college campus. Construct a probability distribution and then find the mean and the standard deviation of the probability distribution.

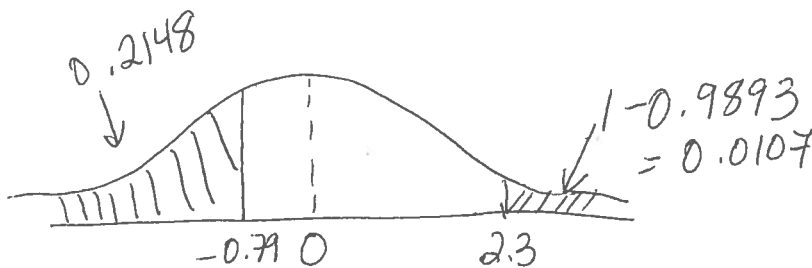
Cups	Students	$P(x)$	$x - \bar{x}$	$(x - \bar{x})^2$	$(x - \bar{x})^2 \cdot P(x)$
0	41	$41/120 = 0.342$	-1.3	1.69	0.57798
1	36	$36/120 = 0.3$	-0.3	0.09	0.027
2	20	0.167	0.7	0.49	0.08183
3	16	0.133	1.7	2.89	0.38437
4	7	0.058	2.7	7.29	0.42282
	<u>120</u>				<u>1.494</u>

$$\bar{x} = \sum x \cdot P(x) = 0 \cdot 0.342 + 1 \cdot 0.3 + 2 \cdot 0.167 + 3 \cdot 0.133 + 4 \cdot 0.058 = 1.265 \approx 1.3$$

$$\sigma = \sqrt{\sum (x - \bar{x})^2 \cdot P(x)} = \sqrt{1.494} = 1.222$$

Mean 1.3 (1.27 OK)  
 Standard deviation 1.2

6. (10 pts) Find:  $P(z < -0.79 \text{ or } z > 2.30)$



Answer 0.2255

$$P = 0.2148 + 0.0107 = 0.2255$$