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MATH 1180
Midterm II

Do all problems. One page of notes, no calculators, no cheating.

1. (25 points) 40% of schools have science clubs in order to improve their chances of having at least one student win a coveted prize at the Science Fair. Those with clubs have at least one student earn a prize with probability 0.8, while those without earn a prize with probability 0.3.
- What is the probability that at least one student from a school wins a prize?
 - What is the probability that a school that wins a prize has a science club?
 - Suppose schools have an equal number of boys and girls, all of whom MUST enter the Science Fair. At a school with a science club, at least one girl gets a prize with probability 0.5, and at least one boy gets a prize with probability 0.4. At least how many schools with science clubs have more than one prize winner?

a. Let W be the event: school had at least one winner
 C be the event: school has a science club

$$Pr(W|C) = 0.8, Pr(W|no C) = 0.3$$

$$Pr(W) = Pr(W|C)Pr(C) + Pr(W|no C)Pr(no C) \\ = (0.8)(0.4) + (0.3)(0.6) = 0.32 + 0.18 = 0.5$$

b. $Pr(C|W) = \frac{Pr(W|C)Pr(C)}{Pr(W)} = \frac{(0.8)(0.4)}{0.5} = 0.64$

c. For there to be 0.8 of schools that get a prize, there must be at least 0.1 which overlap girls and boys, because $0.4 + 0.5 - 0.8 = 0.1$.

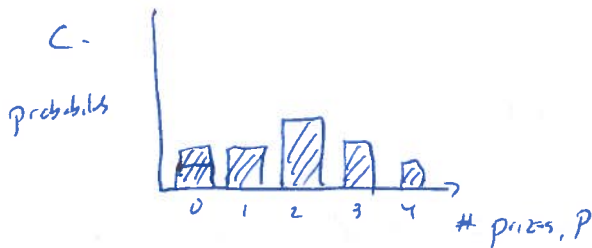
2. (25 points) At the Science Fair, no school is allowed to bring home more than four prizes. Suppose that schools get one prize with probability 0.2, two prizes with probability 0.3, three prizes with probability 0.2, and four prizes with probability 0.1. (These numbers do **not** necessarily match those in #1).

- What is the probability of a school receiving no prizes?
- Write these data in terms of a random variable.
- Sketch a graph illustrating the data.
- What is the expected number of prizes?
- What is the variance?

a.

Number of prizes	Probability
0	? \rightarrow must be 0.2 to add up to 1
1	0.2
2	0.3
3	0.2
4	0.1

- b. Let $P =$ number of prizes.
- $$\Pr(P=0) = 0.2$$
- $$\Pr(P=1) = 0.2$$
- $$\Pr(P=2) = 0.3$$
- $$\Pr(P=3) = 0.2$$
- $$\Pr(P=4) = 0.1$$



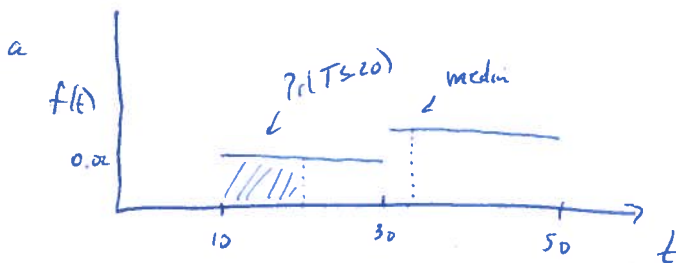
- d. ~~This is symmetric, so it must be $E(P) = 2$ days~~
- Or: $E(P) = 0 \cdot 0.2 + 1 \cdot 0.2 + 2 \cdot 0.3 + 3 \cdot 0.2 + 4 \cdot 0.1 = 1.8$
- e. $E(P^2) = 0 \cdot 0.2 + 1 \cdot 0.2 + 4 \cdot 0.3 + 9 \cdot 0.2 + 16 \cdot 0.1 = 4.8$
- $$V(P) = E(P^2) - E(P)^2 = 4.8 - 1.8^2 = 4.8 - 3.24 = 1.56$$

3. (25 points) Different kids spend different amounts of time working on their projects, described by the p.d.f.

$$f(t) = \begin{cases} 0.02 & \text{if } 10 \leq t \leq 30 \\ 0.03 & \text{if } 30 < t \leq 50 \end{cases}$$

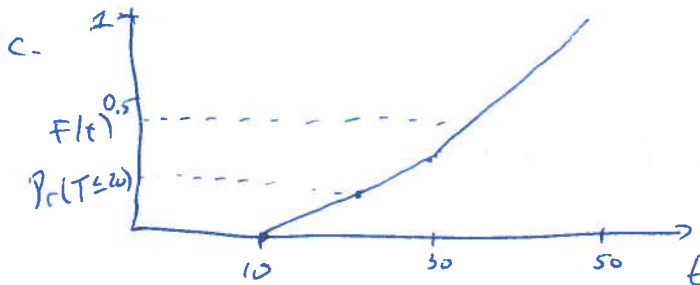
where t is measured in hours.

- Graph this function.
- Check that it is an honest p.d.f.
- Sketch a graph of the c.d.f.
- Find the probability a student spends more than 20 hours. Indicate this value on your graphs.
- Find the median. Indicate this value on your graphs.
- Extra Credit:** What's a good idea for a science fair project on probability?



- b. Always positive

$$\text{Area is } (0.02)(20) + (0.03)(20) = 0.4 + 0.6 = 1$$



d. $\Pr(T > 20) = 1 - \Pr(T \leq 20) = 1 - 10 \cdot 0.02 = 0.8$

e. At median, $\Pr(T \leq T_m) = \Pr(T > T_m) = 0.5$

We know $T_m > 30$ because $F(30) = 0.4$

We can find the even above T_m as $(50 - T_m)(0.03) = 0.5$

$$50 - T_m = \frac{0.5}{0.03} = 16.67$$

$$T_m = 33.33$$

4. (25 points) Schools that win at least one prize in one year become overconfident and win with probability 0.3 in the next year. Those that fail to win at least one prize respond to the resulting public humiliation by winning with probability 0.8 in the next year. (These numbers do **not** necessarily match those in #1 or #2).
- Write these facts in mathematical notation.
 - Sketch a graph of a typical run of wins and losses over time.
 - What fraction of schools will win prizes in the long run?
 - Extra Credit:** If a school wins in one year, what is the probability they also won in the previous year?

a. Let $W_t = \text{wins in year } t$
 $F_t = \text{fails to win in year } t$

$$Pr(W_{t+1} | W_t) = 0.3$$

$$Pr(W_{t+1} | F_t) = 0.8$$



c. Let $g_t = Pr(W_t)$

$$g_{t+1} = Pr(W_{t+1} | W_t) Pr(W_t) + Pr(W_{t+1} | F_t) Pr(F_t)$$

$$= 0.3g_t + 0.8(1-g_t) = 0.8 - 0.5g_t$$

$$g^* = 0.8 - 0.5g^* \Rightarrow 1.5g^* = 0.8, \quad g^* = \frac{0.8}{1.5} = 0.533$$