1\. a $A = 2, 10$ or 25 3 values

$I = 0$ or 1 (uninfected or infected) 2 values

b $Pr(A=2)=0.4$, $Pr(A=10)=0.4$, $Pr(A=25)=0.2$

$Pr(I=1 | A=2) = 0.5$, $Pr(I=1 | A=10) = 0.4$, $Pr(I=1 | A=25) = 0.2$

c. The first part of b is a marginal, the second a conditional

d. $Pr(I=1 \text{ and } A=2) = Pr(I=1 | A=2)\,Pr(A=2) = (0.5)(0.4) = 0.2$

$Pr(I=1 \text{ and } A=10) = Pr(I=1 | A=10)\,Pr(A=10) = (0.4)(0.4) = 0.16$

$Pr(I=1 \text{ and } A=25) = Pr(I=1 | A=25)\,Pr(A=25) = (0.2)(0.2) = 0.04$

		Age 2	Age 10	Age 25	
Infected	0	0.2	0.24	0.16	→ 0.6
	1	0.2	0.16	0.04	→ 0.4
		↓ 0.4	↓ 0.4	↓ 0.2	

e Find marginal of I: $Pr(I=1) = 0.4$

f. $Cov(A, I) = E(AI) - E(A)E(I)$

$E(A) = 2 \cdot 0.4 + 10 \cdot 0.4 + 25 \cdot 0.2 = 9.8$

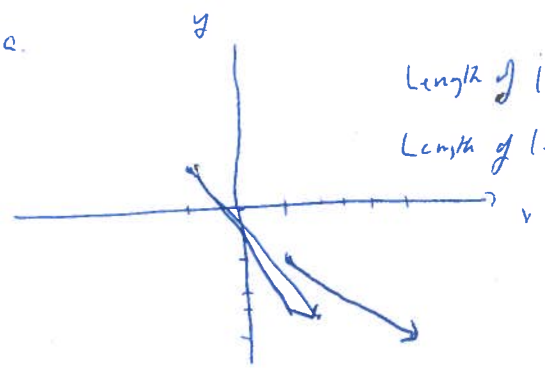
$E(I) = 0.4$

$= (2 \cdot 1 \cdot 0.2 + 10 \cdot 1 \cdot 0.16 + 25 \cdot 1 \cdot 0.04) - (9.8)(0.4)$

$= (0.4) + (1.6) + 1 - (9.8)(0.4) = 3 - (9.8)(0.4) < 0$

Older people less likely to be infected

2a.



Length of $(4,-3) = \sqrt{4^2 + (-3)^2} = 5$ Direction $= \tan^{-1}(-3/4)$

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we don't need to add π because horizontal component is positive

b. End of first vector is $(1,-1) + (4,-3) = (5,-4)$ Length is $r = \sqrt{5^2 + (-4)^2} = \sqrt{41}$, $\theta = \tan^{-1}(-4/5)$

End of second vector is $(-1, 1) + (3,-4) = (2,-3)$ Length is $r = \sqrt{2^2 + (-3)^2} = \sqrt{13}$, $\theta = \tan^{-1}(-3/2)$

3. a. Brutus would become 20% more confident on his own, but loses confidence as ~~Cassius~~ Caesar becomes more confident

Caesar would lose 5% of his confidence daily, as has confidence further sapped by Brutus

b
$$\begin{pmatrix} B_{t+1} \\ C_{t+1} \end{pmatrix} = \begin{pmatrix} 1.2 & -0.2 \\ -0.1 & 0.95 \end{pmatrix}\begin{pmatrix} B_t \\ C_t \end{pmatrix}$$

We have to rewrite the second equation as

$$C_{t+1} = -0.1B_t + 0.95C_t$$

to get the variables in the right order

c
$$\begin{pmatrix} 1.2 & -0.2 \\ -0.1 & 0.95 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1.2\cdot 1 - 0.2\cdot 1 \\ -0.1\cdot 1 + 0.95\cdot 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0.85 \end{pmatrix} = \begin{pmatrix} B_1 \\ C_1 \end{pmatrix}$$

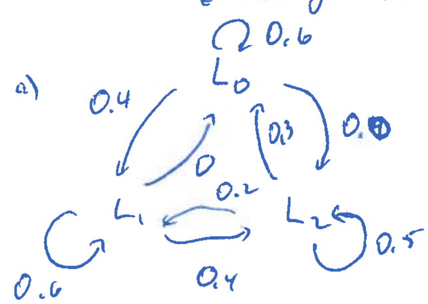
$$\begin{pmatrix} 1.2 & -0.2 \\ -0.1 & 0.95 \end{pmatrix}\begin{pmatrix} 1 \\ 0.85 \end{pmatrix} = \begin{pmatrix} 1.2\cdot 1 - 0.2\cdot 0.85 \\ -0.1\cdot 1 + 0.95\cdot 0.85 \end{pmatrix} = \begin{pmatrix} 1.03 \\ 0.7075 \end{pmatrix} = \begin{pmatrix} B_2 \\ C_2 \end{pmatrix}$$

d
$$\begin{pmatrix} 1.2 & -0.2 \\ -0.1 & 0.95 \end{pmatrix}\begin{pmatrix} 1.2 & -0.2 \\ -0.1 & 0.95 \end{pmatrix} = \begin{pmatrix} 1.2\cdot 1.2 - 0.2\cdot(-0.1) & 1.2\cdot(-0.2) - (0.2)(0.95) \\ 1.2(-0.1)\,(0.1)(0.95) & (-0.1)(-0.2) + 0.95\cdot 0.95 \end{pmatrix}$$

$$= \begin{pmatrix} 1.46 & -0.43 \\ -0.215 & 0.9225 \end{pmatrix}$$

e. I think B_t will grow and C_t will decline.

4. a)



b)
$$\begin{pmatrix} L_0(t+1) \\ L_1(t+1) \\ L_2(t+1) \end{pmatrix} = \begin{pmatrix} 0.6 & 0 & 0.3 \\ 0.4 & 0.6 & 0.2 \\ 0 & 0.4 & 0.5 \end{pmatrix}\begin{pmatrix} L_0(t) \\ L_1(t) \\ L_2(t) \end{pmatrix}$$

c) Equilibrium:
$L_0^* = 0.6L_0^* + 0.3L_2^* \Rightarrow 0.4L_0^* = 0.3L_2^* \sim L_0^* = \frac{3}{4}L_2^*$

$L_1^* = 0.4L_0^* + 0.6L_1^* + 0.2L_2^*$

$L_2^* = 0.4L_1^* + 0.5L_2^* \Rightarrow 0.5L_2^* = 0.4L_1^* \Rightarrow L_1^* = \frac{5}{4}L_2^*$

$L_0^* + L_1^* + L_2^* = 1 \sim \frac{3}{4}L_2^* + \frac{5}{4}L_2^* + L_2^* = 1 \sim L_2^* = 1/3 \Rightarrow L_0^* = \frac{1}{4},\ L_1^* = \frac{5}{12}$

d. Expected number: $0\cdot\frac{1}{4} + 1\cdot\frac{5}{12} + 2\cdot\frac{1}{3} = \frac{5}{12} + \frac{2}{3} = \frac{13}{12}$