MATH 1180
Midterm II

Do all problems. One page of notes, no calculators, no cheating.

1. (25 points) A professor is thrilled to find two markers in the classroom on the rare day when she happens to have forgotten her own markers. She knows that every marker works with probability 0.7, and that the second marker will work with probability 0.9 if the first works.

   a. Develop notation to express this as a conditional probability.

   b. What is the probability that both markers work?

   c. Write the probability that the second marker works by using the law of total probability.

   d. Use this to find the probability that the second marker works conditional on the first marker not working.

   e. **Extra Credit:** What is the offical name for a group of lemmings?

2. (25 points) The next day, there are again two markers in the room. The probability that at least one works is 0.7, and the probability that at least one does not work is 0.5.

    a. Find the probability that exactly 1 marker works.

    b. Find the whole probability distribution (if you can't get this, make something up for the next two parts).

    c. Find the cumulative distribution.

    d. What is the expected number of working markers?

    e. Extra Credit: What is the total number of syllables in a haiku?

3. (25 points) Markers don't simply just work or not, but they have different qualities. Suppose marker quality Q is described by the p.d.f.

$$f(q) = 1.5(q - 1)^2$$

where $0 \leq q \leq 2$.

    a. Check that this function satisfies the requirements for a p.d.f.

    b. Graph it.

    c. Sketch a graph of the c.d.f.

    d. Write the expression you would use to find $\text{Var}(Q)$.

    e. Find the mode of Q.

    f. **Extra Credit:** What is the probability that a randomly chosen integer is a multiple of 3 but not a multiple of 5?

4. (25 points) The university administration is not sympathetic to complaints about markers that don't work, and claims that markers will recover overnight if you just put the caps on. In fact, a marker that doesn't work on one day recovers and works the next day with probabiliy 0.2, but a marker that works one day does not work on the next with probability 0.6.

    a. Write these facts in mathematical notation.

    b. What is the probability that a marker that doesn't work on one day does work two days later?

    c. What fraction of markers will work in the long run?

    d. **Extra Credit:** What would you call a Markov chain describing markers?