REU Project Proposal
Applications of GARCH–type processes

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The GARCH (1,1) model was originally introduced by Bollerslev (1986) and Engle (1982) in order to model the fluctuations of the variances of the time series data. They observed that there is significant variation between the variances of log returns of stocks. Since the aforementioned stock values do not have constant variance, the volatility of the stock market can be modeled using the GARCH (1,1) model. It is assumed that the log returns will satisfy the following equations

$$y_k = \sigma_k \epsilon_k, \quad -\infty < k < \infty \tag{0.1}$$

and

$$\sigma_k^2 = \omega + \alpha y_{k-1}^2 + \beta \sigma_{k-1}^2, \quad -\infty < k < \infty, \tag{0.2}$$

where (ω, α, β) is the parameter of the process. It is assumed that the errors (innovations) $\epsilon_k, -\infty < k < \infty$ are independent identically distributed random variables. Nelson (1990) found the necessary and sufficient condition for the existence of $(y_k, \sigma_k^2), -\infty < k < \infty$. Lumsdaine (1996) used the quasi-maximum likelihood method to estimate the parameters.

In the GARCH(1,1) model, both the positive and negative returns were given equal weight. The GARCH (1,1) model was later revised by Glosten, Jagannathan and Runke (1993) to create the GJR–GARCH model. In this revised model, the negarive returns are given larger weight. In other words, in the GJR-GARCH(1, 1), (0.2) is replaced by

$$\sigma_k^2 = \omega + \alpha_1 y_{k-1}^2 I\{y_{k-1} < 0\} + \alpha_2 y_{k-1}^2 I\{y_{k-1} \geq 0\} + \beta \sigma_{k-1}^2, \tag{0.3}$$

$-\infty < k < \infty$,where $\boldsymbol{\theta} = (\omega, \alpha_1, \alpha_2, \beta)$ is the parameter of the process.

One of the goals for my project is to use simulations, probably in SAS or another statistical package, to study the properties of the estimator. Furthermore, I want to fit the data to the GARCH(1,1) model; and study the fit. I want to determine whether GARCH(1,1) or GJR–GARCH(1,1) gives a better fit the the data. I want to use a data set containing IBM data to accomplish this.

References

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