Optimal Drawing Strategy for Deuces Wild Video Poker

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Abstract

Video poker, an extremely popular computerized version of the traditional game of poker, offers many opportunities for mathematical analysis. Often, an optimal strategy exists which can, in the long run, assure a player of winning more than he bet. As part of a summer undergraduate research project, Professor Ethier and I chose to analyze the "Deuces Wild" variant of video poker and to find its optimal strategy. Choosing this particular type of video poker was motivated by its exceptional popularity as well as the added challenge posed to the analysis by the presence of wild cards. Since this analysis has been done before[**?**], albeit with with much less precision than the present analysis, it provided a good problem for an undergraduate project since the results could be verified.

1 Introduction

Video poker games are typically based on traditional five card draw poker. Many variants of video poker exists, and each have a particular set of rules. For "Deuces Wild," the twos are wild, in that they can take the place of any other card in a hand. A game of video poker is begun by placing a bet, usually one to five units of currency (such as quarters or dollars). The machine then deals five cards from a randomized deck of 52 cards. The player can then choose which cards to keep and the machine replaces discarded cards from the top of the deck. A bet is won when the resulting hand is a "paid hand" illustrated below.

Table 1: Paid hands, examples, and their payoffs for a one unit bet

Paid Hand	Example	Payoff
Natural Royal Flush	10♣J♣Q♣K♣A♣	800
Four Decues	4♡2♡2♣2♠2♢	200
Wild Royal Flush	10♡J♡K♡2♠2♢	25
Five of a Kind	4♠4♣4♡4♢2♣	15
Straight Flush	3♣4♣5♣6♣7♣	9
Four of a Kind	6♣6♡6♠6♢J♣	5
Full House	4♣4♢4♡8♠8♡	3
Flush	3♡8♡7♡10♡A♡	2
Straight	4♣5♠6♠7♡8♢	2
Three of a Kind	8♣8♡8♢J♣3♣	1
Other	3♡9♡7♠J♢A♣	0

After winnings are paid (or not paid) and another bet is placed, the machine creates a new random deck and deals another five cards. It should be noted that for the 800 for 1 payoff, a five unit bet is usually required.

2 Method

The optimal strategy of a video poker game is the combination of the optimal ways to play each of the $\binom{52}{5}$ possible hands. The optimal play for a hand is found by computing the expected value for each of the 2^5 possible ways to play the hand and taking the play with the highest expected value. This is complicated by the presence of "penalty cards" which, when discarded, affect the chance of getting a certain paid hand, and therefore the expected value of the hand. It becomes obvious that doing this analysis entirely by hand is an extremely difficult problem. Using a computer, though, it becomes much easier.

To complete this on a computer, I devised an algorithm which would find the optimal play for all of the possible hands. For each possible play of each hand, the program would replace the discards from the remaining 47 cards in the deck and count how many of each paid hand it encountered. Using these quantities, it would divide the number of instances of the paid hands by $\binom{47}{j}$, j=number of cards drawn. This became the probability of getting

that particular paid hand. Then, multiplying the probability of getting a paid hand by the associated payoff, and then adding these values together, the program arrived at the expected value. This was then repeated 31 more times and the greatest value found was kept as the optimal play. This process was then repeated for each of the approximately 2.6 million possible hands.

Done improperly (see below), even a fast computer would take a long time to finish this problem. A major optimization which can be done, though, is to pare down the number of possible hands. This is feasible since there are symmetries in the suits. For instance, there are four possible ways to get a particular straight flush, with no wilds: one for each suit. The expected value calculation only needs to be done for one suit, since the other three have equal expected values. By figuring out these suit-based symmetries the total number of distinct possible hands goes from about 2.6 million to around 130 thousand.

The program I wrote took about 10 seconds to analyze each hand on an SGI R5000 Indy. So to completely finish the program would take 15 days. This was most likely due to lack of programming experience on my part. Fortunately, I found some source code by Johann Ruegg, from the rec.gambling archive[**?**], which could solve the problem in about 50 minutes on my Indy and about 10 on a Pentium III-based Linux PC. After spending some time learning how the program worked, I set about modifying it to give me the output I wanted. Then, I was able to analyze the output to get the final strategy. Incidentally, the program I found used the same basic algorithm which I had come up with earlier, but was put in to practice much more efficiently.

3   Strategy

To analyze the data, I sorted all of the optimally played hands by their expected value. Hands with similar optimal plays became grouped together since the deduction from the expected value due to the presence of a penalty card was usually quite small. There were still a very large number of hands to deal with, though, so I removed the hands with duplicated expected values. This yielded 8000 distinct expected values. I further reduced the list by removing all of the hands in which all non-deuce cards were removed. I used the remaining hands as the basis for my strategy.

Also, several terms which are used in this section should be defined here.

An *open straight* is a partial straight whose cards are grouped together sequentially and can be completed in both directions. An *inside straight* is a partial straight whose cards may contain gaps, or can only be completed in one direction. A *made pay* is a hand which contains a paid hand, but may contain other cards which can be discarded without reducing the expected return. For example, a four of a kind with a fifth unrelated, non-deuce card. A *pat hand* is a paid hand which, if any cards are discarded from it, will go down in expected return.

Below are the rules, with exceptions, for optimally playing this game. Rules marked with a * deviate from those in Paymar[**?**].

Hold all deuces.
Discard all non-deuce cards except draw to the hands shown below.

no deuces

- Any made pays, except:

    - Four suited royals

- Sets of two or more suited royals, except:

    - A (K,Q), (K,J), or (K,T) with one or more penalty cards; i.e. a 9-A of a different suit that the (K,X) (penalty against a straight), or 3-8 of the same suit (penalty against a flush)

- Any three or four card straight flush, except:

    - Three-card inside straight flush with a penalty card*–except (3,4,6) and (3,5,6) with one penalty card; i.e. a straight-completing card of a different suit (penalty against a straight), or another card of a different suit (penalty against a flush)

- Any inside or open four-card straight, except:

    - Four-card inside straight with an ace-low, e.g. (A,3,4,5)

- Any pair–throw away a second pair, if present

- Any four suited cards

one deuce

- Made pays four of a kind or better

- Sets of two or more suited royals, except:

  - (A,K) and (4,5), (4,6) or (3, any 4-9), except (3,8) of different suits, or a penalty card*

  - (A,Q) and (3,4), (3,5) or (3,6), or a penalty card*

  - (A,J) and (3,4), (3,5) of the same suit, or a penalty card*

  - (A,10) and (3,4) of the same suit, or a penalty card*

  - Penalty cards for this case are: a royal of a different suit than the (A,X) set (penalty against a straight), or a 3-9 of the same suit as the (A,X) set (penalty against a flush)

- Three of a kind

- Any three or four card straight flush (including deuce), except:

  - Open three-card straight-flush with deuce and (4,5) and (5,6)

  - Any inside three-card straight-flush

two deuces

- Pat hands straight flush or better

- Four of a kind

- Sets of two suited royals

- Any suited, consecutive, two-card sequence, except:

  - (3,4), (4,5) or (5,6)

three deuces

- Pat wild royal flush

- Pat five of a kind, except when the non-deuces are (9,9) or lower* (details below)

four deuces

- Keep hand as is

An interesting and counter-intuitive example is the pat five of a kind with three deuces. The hand looks as though discarding any pair would give the same expected value. However, if the pair is (9,9) or lower, discarding the pair gives the player a chance for a wild royal flush. If the pairs are (10,10) or higher, then discarding them actually reduces the chance of getting the wild royal flush. Here are the computations:

$E[H|\text{Five of a kind}] = 15$

For (2,2,2) and (9,9) or lower, $E[H|\text{discard 9s}]$:

$$\frac{46 \cdot 200 + 40 \cdot 25 + 67 \cdot 15 + 108 \cdot 9 + (1035 - 40 - 67 - 108) \cdot 5}{\binom{47}{2}} \approx 15.0574\ldots$$

For (2,2,2) and (10,10) or higher, $E[H|\text{discard 10s}]$:

$$\frac{46 \cdot 200 + 32 \cdot 25 + 67 \cdot 15 + 116 \cdot 9 + (1035 - 32 - 67 - 116) \cdot 5}{\binom{47}{2}} \approx 14.9389\ldots$$

So the correct way to play this hand is to throw away a pair (9,9) or lower and to keep a pair of (10,10) or higher.

4   Additional

I also found the probability of getting each paid hand, which can be used to calculate the total expected return of this game. The probabilities can be found by dividing X by $\binom{52}{5}\binom{47}{5}5$.

Table 2: Paid hands and their probabilities

Hand	X	Probability
Natural Royal Flush	440 202 756	.00002208386
Four Deuces	4 060 462 824	.0002037032
Royal Flush	35 796 957 696	.001795843
Five of a Kind	63 818 309 856	.003201604
Straight Flush	83 218 233 480	.004174849
Four of a Kind	1 294 427 430 576	.06493817
Full House	423 165 297 240	.02122914
Flush	335 108 390 364	.01681154
Straight	1 116 335 570 916	.05600375
Three of a Kind	5 675 175 572 112	.2847093
Nothing	10 901 684 089 380	.5469100

So, using this to calculate the expected return, we get:

$$\frac{20085114000432}{\binom{52}{5}\binom{47}{5}5} = \frac{32187682693}{31944279675} \approx 1.007619612\ldots$$

In Paymar[**?**], the final result was given as 100.762%, which has apparently been rounded. This table is also useful for doing quick expected return calculations for machines which have a different pay schedule than the one given in Table 1.

References

[1] Paymar, Dan, *Video Poker—Optimum Play*, 1999. ConJelCo, Pittsburgh.

[2] rec.gambling archive, ftp://ftp.csua.berkeley.edu/pub/rec.gambling/