Calculus III
Practice Problems 12

1. A fluid has density 3 and velocity field $\mathbf{V} = 4x\mathbf{I} + 3z\mathbf{J} - z\mathbf{K}$. Find the flux of the fluid out of the ball centered at the origin and of radius 4 through its boundary.

2. Let P be the parabolic cup $z = x^2 + y^2$ lying over the unit disc in the xy-plane. Let $\mathbf{F}(x,y,z) = y\mathbf{I} - x\mathbf{J} + \mathbf{K}$. Calculate

$$\int\int_P \text{curl } \mathbf{F} \cdot \mathbf{N} dS \, .$$

3. Evaluate $\int\int_S \mathbf{F} \cdot \mathbf{N} dS$, where $\mathbf{F}(x,y,z) = x\mathbf{I} + y\mathbf{J} + z\mathbf{K}$ and S is the part of the paraboloid $z = 4 - x^2 - y^2$ which lies above the xy-plane.

4. Evaluate $\int\int_S \sqrt{1 + x^2 + y^2} dS$ where S is the surface given parametrically by

$$\mathbf{X}(s,t) = s\cos t\mathbf{I} + s\sin t\mathbf{J} + t\mathbf{K} \, , \quad 0 \leq s \leq 5, 0 \leq t \leq \pi/2 \, .$$

5. Let S be the part of the plane $2x + y + 3z = 12$ which lies in the first octant, oriented upward. Let the boundary ∂S of S be oriented so that S is to its left. Given the vector field $\mathbf{F} = 3x\mathbf{I} + \mathbf{J} + y\mathbf{K}$, find $\int_{\partial S} \mathbf{F} \cdot d\mathbf{X}$.

6. Let B^+ be the half-ball $B : x^2 + y^2 + z^2 \leq 1, \, z \geq 0$. Let $\mathbf{F}(x,y,z) = x\mathbf{I} + y\mathbf{J} + \mathbf{K}$. Let H be the hemisphere bounding B^+ above: $H : x^2 + y^2 + z^2 = 1, \, z \geq 0$. Calculate the flux of $\mathbf{F}$ from B^+ across H.

7. Let $\mathbf{F} = x^2\mathbf{I} + y^2\mathbf{J} + z^2\mathbf{K}$. Calculate the flux of $\mathbf{F}$ out of the sphere S of radius 3.

8. Let P be the piece of the plane $2x + y + 3z = 12$ which lies in the first octant, and let $\mathbf{F} = 3x\mathbf{I} + \mathbf{J} + y\mathbf{K}$. Calculate the flux of $\mathbf{F}$ through P from below.