2210-90 Exam 1

Summer 2013

Name ____

Instructions. Show all work and include appropriate explanations when space is provided. Correct answers unaccompanied by work may not receive full credit. Page 5 is blank in case you need extra paper. Please circle your final answers.

1. (16pts) Consider the vectors $\mathbf{u} = \langle 6, 0, 2 \rangle$ and $\mathbf{v} = \langle -1, 7, 3 \rangle$. Find
   (a) (2pts) $\mathbf{v} - 2\mathbf{u}$
   (b) (2pts) $||\mathbf{u}||$
   (c) (2pts) The unit vector which points in the same direction as $\mathbf{u}$
   (d) (2pts) $\mathbf{u} \cdot \mathbf{v}$
   (e) (1pt) Are $\mathbf{u}$ and $\mathbf{v}$ orthogonal? Circle one: YES NO
   (f) (3pts) $\mathbf{u} \times \mathbf{v}$
   (g) (4pts) Two of the following quantities are zero (or the zero vector). Which ones? Circle two letters.

   A. $\mathbf{u} \cdot (\mathbf{u} \times \mathbf{v})$ B. $\mathbf{u} \times \mathbf{u}$ C. $\mathbf{u} \cdot \mathbf{u}$ D. $\mathbf{v} \times (\mathbf{u} \times \mathbf{v})$

2. (7pts) Find an equation of the plane consisting of all points that are equidistant from the points $P = (1, 0, -1)$ and $Q = (3, 2, 1)$.

3. (17pts) Suppose a particle's position at time t is given by the curve

$$\mathbf{r}(t) = (\cos t + t \sin t)\mathbf{i} + 4t^2\mathbf{j} + (\sin t - t\cos t)\mathbf{k}.$$

For this problem, it is helpful if you remember the trig identity $\sin^2 t + \cos^2 t = 1$.

(a) (2pts) Find the velocity $\mathbf{v}(t)$ of the particle at time t.

(b) (3pts) Find the arc length of the curve between times $t = 0$ and $t = 2$.

(c) (2pts) Find the acceleration $\mathbf{a}(t)$ of the particle at time t.

(d) (2pts) Find the unit tangent vector $\mathbf{T}(t) = \frac{\mathbf{v}(t)}{||\mathbf{v}(t)||}$.

(e) (3pts) Find the principal unit normal vector $\mathbf{N}(t) = \frac{\mathbf{T}'(t)}{||\mathbf{T}'(t)||}$.

(f) (5pts) Find the curvature $\kappa(t)$ of the particle's path at time t.

4. (5pts) Suppose the acceleration of a particle is given by

$$\mathbf{a}(t) = \langle 2t, t + \sin t, e^{-t} \rangle.$$

If the particle's initial velocity is $\mathbf{v}(0) = \langle 2, -3, 1 \rangle$, what is the velocity of the particle at time t?

5. (14pts) Match the equation with the type of surface it determines by writing the appropriate capital letter (**A-G**) in the provided blank. Each letter should be used exactly once.

______	$x^2 + y^2 + z^2 = 1$	**A** Elliptic Paraboloid
______	$x^2 + z^2 - y^2 = 1$	**B** Ellipsoid
______	$x^2 + 2y^2 + 3z^2 = 1$	**C** Hyperboloid of one sheet
______	$x^2 - 2y^2 - z = 0$	**D** Hyperboloid of two sheets
______	$y = x + 3z - 7$	**E** Hyperbolic Paraboloid
______	$x^2 + 2y^2 - z = 0$	**F** Plane
______	$z^2 - x^2 - y^2 = 1$	**G** Sphere

6. (8pts) Match the equation and the description of the surface by writing the appropriate capital letter (**A-D**) in the provided blank. Each letter should be used exactly once.

(a) ______ In cylindrical coordinates, the surface $z = r^2$.
(b) ______ In cylindrical coordinates, the surface $r^2 + z^2 = 4$.
(c) ______ In spherical coordinates, the surface $\rho = 2\cos\phi$.
(d) ______ In spherical coordinates, the surface $\theta = \frac{3\pi}{4}$.

A a sphere centered at the origin.
B a half-plane.
C a paraboloid
D a sphere centered at the point $(0, 0, 1)$ in Cartesian coordinates.

7. (9pts) Convert between Cartesian, cylindrical, and spherical coordinates as indicated. Please simplify as much as possible.

(a) Find the cylindrical coordinates of the point with Cartesian coordinates $(-1, 1, 3)$

$r =$ ____________ $\theta =$ ____________ $z =$ ____________

(b) Find the spherical coordinates of the point with Cartesian coordinates $(1, \sqrt{3}, -2)$

$\rho =$ ____________ $\theta =$ ____________ $\phi =$ ____________

(c) Find the Cartesian coordinates of the point with cylindrical coordinates $(2, \frac{\pi}{6}, \pi)$

$x =$ ____________ $y =$ ____________ $z =$ ____________

8. (12pts) Evaluate the following limits. If they do not exist, write 'DNE' and explain why.

(a) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{e^{x^2+y^2}}{1+x^2+y^2}$

(b) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2+y}{x^2-y}$

(c) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^3y}{x^2+y^2}$ **Hint:** Use polar coordinates.

(d) $\displaystyle\lim_{h\to 0} \frac{\sin((x+h)y)-\sin(xy)}{h}$ **Hint:** Think derivative.

9. (12pts) Consider the function

$$f(x,y) = xy\cos(x^2).$$

Compute the following partial derivatives:

(a) $f_x(x,y) =$

(b) $f_y(x,y) =$

(c) $f_{yy}(x,y) =$

(d) Find $\nabla f(0,2)$. That is find the gradient of f at the point $(0,2)$.