Math 1210 #21
Solving Equations Numerically

Three numeric methods for solving an equation numerically:

1. Bisection Method
2. Newton's Method
3. Fixed-point Method

Bisection Method Algorithm

Let $f(x)$ be a continuous function and let a_1 and b_1 be numbers satisfying $a_1 < b_1$ and $f(a_1) \cdot f(b_1) < 0$.

Let E denote the desired bound for the error $|r - m_n|$.

Pros:

Cons:

Repeat steps 1 to 5 for $n = 1, 2, \ldots$ until $h_n < E$.

1. Calculate $m_n = \frac{(a_n + b_n)}{2}$.
2. Calculate f(m_n) and if f(m_n) $= 0$ then STOP.
3. Calculate $h_n = \left|\frac{b_n - a_n}{2}\right|$ (for error testing).
4. If $f(a_n) \cdot f(m_n) < 0$, then set $a_{n+1} = a_n$ and $b_{n+1} = m_n$.
5. If $f(a_n) \cdot f(m_n) > 0$, then set $a_{n+1} = m_n$ and $b_{n+1} = b_n$.

EX 1

Approximate the real root to 2 decimal places. f(x)=x4+5x3+1 on [-1,0]

Newton's Method Algorithm

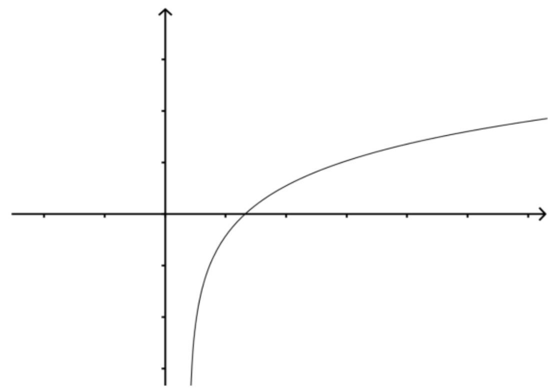
Let $f(x)$be a differentiable function and let x_1 be an initial approximation to the root, r, of $f(x) = 0$. Let E denote a bound for the error $|r - x_n|$.

Repeat the following step for $n = 1, 2, \ldots$ until $|x_{n+1} - x_n| < E$.

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Pros:

Cons:



EX 2

Use Newton's method to approximate a root of $7x^3 + 2x - 5 = 0$ to 5 decimal places.

Warning on Newton's Method: