Solutions to practice exam 2 for Math 1060-90 Online Trigonometry

Formulas and identities on the front of exam:

A few formulas:

Law of cosines: $c^2=a^2+b^2-2ab\cos C$

$\text{Area} = \frac{1}{2}ab\sin C$

Sum diff:

$\sin(u+v) = \sin u\cos v + \cos u\sin v$

$\cos(u+v) = \cos u\cos v - \sin u\sin v$

$\tan(u+v) = \dfrac{\tan u + \tan v}{1-\tan u\tan v}$

Double angle:

$\sin 2u = 2\sin u\cos u$

$\cos 2u = 2\cos^2 u - 1 = 1 - 2\sin^2 u$

$\tan 2u = \dfrac{2\tan u}{1-\tan^2 u}$

half angle:

$\sin\frac{u}{2} = \pm\sqrt{\dfrac{1-\cos u}{2}}$ $\quad \cos\frac{u}{2} = \pm\sqrt{\dfrac{1+\cos u}{2}}$ $\quad \tan\frac{u}{2} = \dfrac{1-\cos u}{\sin u}$

Formulas and identities you are expected to know:

Reciprocal identities: $\sec x = \dfrac{1}{\cos x}$

ratios: $\tan x = \dfrac{\sin x}{\cos x}$

Pythagorean Identities

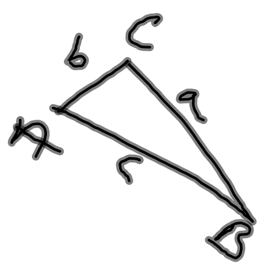
$$\frac{\sin^2 x}{\cos^2 x} + \frac{\cos^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$$

$$\tan^2 x + 1 = \sec^2 x$$

Law of sines

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

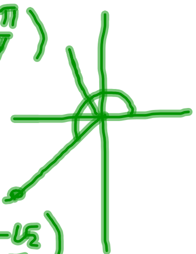


1. a. sin 5π/8

Use half-angle formula

$\frac{5\pi}{8} = \frac{1}{2}\left(\frac{5\pi}{4}\right)$

$\sin\frac{\frac{5\pi}{4}}{2} = +\sqrt{\frac{1-\cos\frac{5\pi}{4}}{2}}$



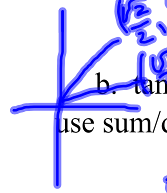
$\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$

$\sqrt{\frac{1+\frac{\sqrt{2}}{2}}{2}}\left(\frac{2}{2}\right)$

$\sqrt{\frac{2+\sqrt{2}}{4}} = \frac{\sqrt{2+\sqrt{2}}}{2}$

b. tan 5π/12

use sum/difference formula



$\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ 1

$\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ $\frac{1}{\sqrt{3}}$

$\tan\left(\frac{3\pi}{12} + \frac{2\pi}{12}\right) = \tan\left(\frac{\pi}{4} + \frac{\pi}{6}\right)$

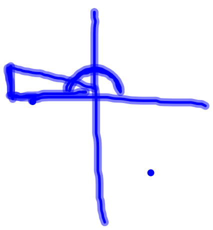
$\frac{\tan\frac{\pi}{4} + \tan\frac{\pi}{6}}{1 - \tan\frac{\pi}{4}\cdot\tan\frac{\pi}{6}}$

$\frac{\sqrt{3}}{\sqrt{3}}\left(\frac{1 + \frac{1}{\sqrt{3}}}{1 - 1\cdot\frac{1}{\sqrt{3}}}\right) = \frac{\sqrt{3}+1}{\sqrt{3}-1}$

Inverse functions! Only 1 answer!

2. a. $\cos^{-1}(-\sqrt{3}/2)$

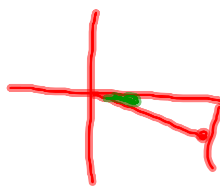
$= \frac{5\pi}{6}$



b. $\tan^{-1}(-1/\sqrt{3})$

$= -\frac{\pi}{6}$

$-\frac{1}{\sqrt{3}}$



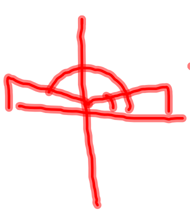
$\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

Radians

3. State all values on the interval [0,2π] for which these are true.

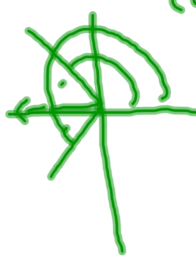
a. sin x = 1/2

$\frac{\pi}{6}, \frac{5\pi}{6}$



b. sec x = - √ 2

$\cos x = -\frac{1}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$



$\frac{3\pi}{4}, \frac{5\pi}{4}$

3. Determine all solutions for this equation on the interval $[0,2\pi)$

$2\sin^2 x - \sin x - 1 = 0$ Hint: factor it first.

$(2\sin x + 1)(\sin x - 1) = 0$

$\sin x = -\frac{1}{2}$ $\sin x = 1$

$x = \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{\pi}{2}$

5. If $\cos\alpha = -3/5$ and $\tan\alpha < 0$ find $\cot\alpha$ and $\sin(2\alpha)$

$\cot\alpha = \frac{-3}{4}$

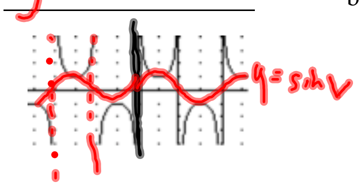
$\sin 2\alpha = 2\sin\alpha\cos\alpha$
$= 2\left(\frac{4}{5}\right)\left(\frac{-3}{5}\right)$
$= \frac{-24}{25}$

Graphing trig functions.

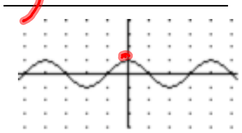
For all graphs on this page x is on the interval and y is on the interval [-4,4]

6. These basic trig function graphs have **no** transformations. Write the equation of each.

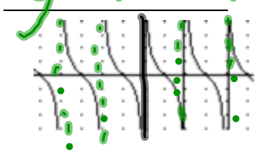
a. $y = \csc x$



b. $y = \cos x$



c. $y = \cot x$



7. Circle all the expressions which have a value of 1:

$(\sin x)(\tan x)$

$\sin x \cdot \frac{\sin x}{\cos x} = \frac{\sin^2 x}{\cos x}$

$\sec^2 x - \tan^2 x$

$\sin^2 x + \cos^2 x = 1$

$\tan^2 x + 1 = \sec^2 x$

$1 = \sec^2 x - \tan^2 x$

$\frac{\cos x}{\sec x}$

$\cos x \cdot \frac{1}{\sec x}$

$\cos x \cdot \cos x$

$\sin^2 x + \cos^2 x = 1$

$\sqrt{(\sin x)(\tan x)(\sec x)}$

$\sqrt{\sin x \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x}}$

$\sqrt{\frac{\sin^2 x}{\cos^2 x}} = \sqrt{\tan^2 x}$

$= \tan x$

$(\tan x)(\cot x)$

$\tan x \frac{1}{\tan x}$

8. Expand and simplify: $(\sin x + \cos x)^2$

$\sin^2 x + 2\sin x\cos x + \cos^2 x$

$1 + 2\sin x\cos x$

$1 + \sin 2x$

9. Verify this identity: $\frac{\sec^2 x - 1}{\sec^2 x} = \sin^2 x$

$1 - \frac{1}{\sec^2 x}$

$1 - \cos^2 x$

$\sin^2 x$

10. Add and simplify: $\frac{\sin \phi}{\cos \phi} + \frac{\cos \phi}{\sin \phi} =$

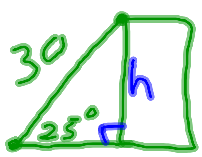
$\frac{\sin x}{\sin x}$, $\frac{\cos x}{\cos x}$

$\frac{\sin^2 x + \cos^2 x}{\sin x \cos x} = \frac{1}{\sin x \cos x}$

$\boxed{\csc x \cdot \sec x}$

You may **use a calculator on these.** Round angles to 1 decimal place and sides to 2.. Draw a sketch, set up the math and solve for the variable. Because you have a calculator, you must be certain to show the math. Show the set-up and steps. Don't forget your units. (feet, degrees, etc.)

11. If a 30 foot wire from the roof of a building to the ground makes a 25º angle with the ground, how high is the building?



✓

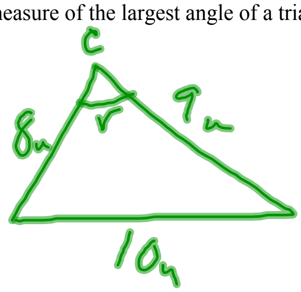
✓ $\sin 25° = \frac{h}{30}$

✓ $h = 30 \sin 25° \approx 12.68 \text{ ft}$

1 pt

12. What is the measure of the largest angle of a triangle with sides 8, 9 and 10. What is the area of the triangle?

uni



Law of cos

$10^2 = 8^2 + 9^2 - 2(8)(9)\cos C$

$100 - 64 - 81 = -144 \cos C$

$\frac{-45}{-144} = \cos C$

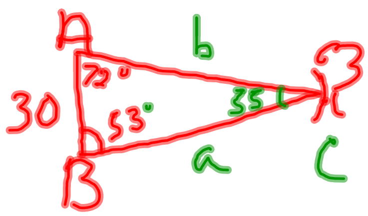
$C = \cos^{-1}\left(\frac{5}{16}\right) \approx 71.8°$

$A = \frac{1}{2} ab \sin C$

$= \frac{1}{2}(8)(9)\sin 71.8°$

$\approx 34.20 \ u^2$

13. Two surveyors stand 30 meters apart on a straight path. They each measure the angle from the path to a straight line between them and a fixed tree. For one the angle is 72 degrees and for the other the angle is 53 degrees. How far away from the tree is each surveyor?



$$\frac{b}{\sin B} = \boxed{\frac{a}{\sin A} = \frac{c}{\sin C}}$$

30 m

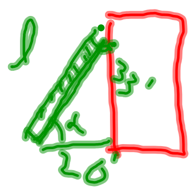
$\sin 53°$ $\sin 72°$ $\sin 55°$

$180 - 72° - 53°$

$$\frac{a}{\sin 72°} = \frac{30}{\sin 55°} \Rightarrow a = \frac{30 \sin 72°}{\sin 55°} \approx 34.83\text{ m}$$

$$\frac{b}{\sin 53°} = \frac{30}{\sin 55°} \Rightarrow b = \frac{30 \sin 53°}{\sin 55°} \approx 29.25\text{ m}$$

14. A ladder is to be placed so the bottom is 20 ft from the wall and the top hits the wall 33 feet up. How long must the ladder be and what angle will it make with the ground?



$$l^2 = 33^2 + 20^2$$

$$l^2 = 1489$$

$$l = \sqrt{1489} \approx 38.59\text{ ft}$$

$$\alpha = \tan^{-1}\left(\frac{33}{20}\right) \approx 58.8°$$