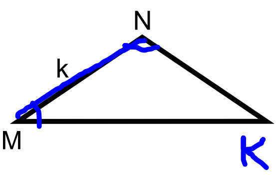
Trig 3.2 ~ The Law of Cosines

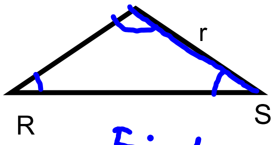
* Prove the law of cosines.
* Use the law of cosines to solve for parts of a triangle.
* Use the law of sines and the law of cosines to solve for parts of a triangle.
* Solve real life problems using these laws.
* Use two ways to find the area of a triangle.

Remembering Geometry Congruence Theorems:

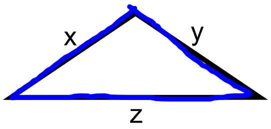
ASA



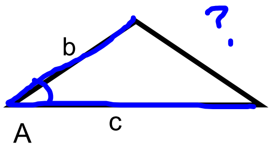
AAS



SSS



SAS



$\angle M + \angle N + \angle K = 180°$

Law of Sines.

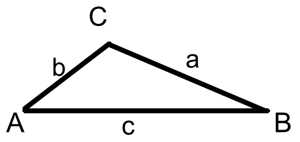
Find 3^{rd} angle.

law of Sines.

Law of Cosines.

SSA - Ambiguity

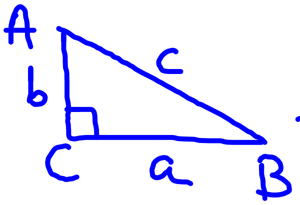
Law of Cosines: In any triangle, ABC with sides a, b, c:



$$\boxed{c^2} = a^2 + b^2 - (2ab)\boxed{\cos C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

It looks like the Pythagorean theorem!

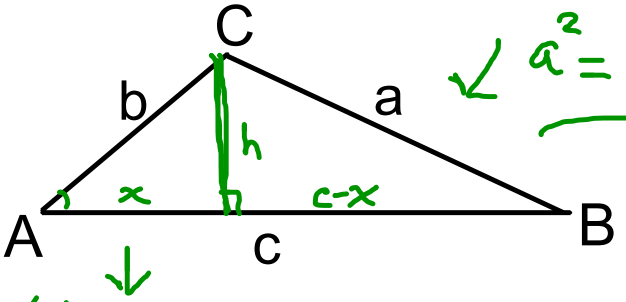


$$= \quad c^2 = a^2 + b^2 - 2ab \cos C$$

$$\cos C = \cos 90^\circ = 0$$

PROOF: Given: Δ ABC with sides a, b, c

Prove: $a^2 = b^2 + c^2 - (2bc)\cos A$



✓ $\underline{a^2 = h^2 + (c-x)^2}$

✓ $h = b\sin A$

✓ $x = b\cos A$

$a^2 = (b\sin A)^2 + (c - b\cos A)^2$

$a^2 = \underline{b^2\sin^2 A} + c^2 - 2bc\cos A + \underline{b^2\cos^2 A}$

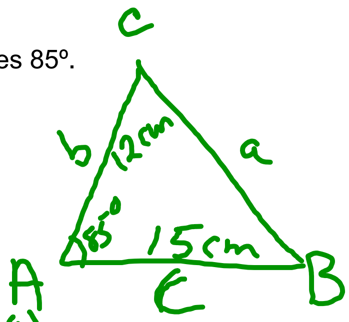
$a^2 = b^2\underline{(\sin^2 A + \cos^2 A)} + c^2 - 2bc\cos A$

$\underline{a}^2 = b^2 + c^2 - 2bc\,\underline{\cos A}$

Example 1 SAS:

Triangle ABC has c = 15 cm, b = 12 cm and ∠A measures 85º.
Solve for the remaining three parts of the triangle.

- *Draw a picture.
- *Label parts.
- *Determine which law to use.
- *Solve.



$a^2 = 12^2 + 15^2 - 2(12)(15)(\cos 85°)$

$a^2 \approx \underline{337.62}$ ——
ans

$\sqrt{ans}$

$\boxed{a = 18.37 \text{ cm}}$

Find ∠B

$\frac{\sin B}{12} = \frac{\sin 85}{18.37}$

$\boxed{\angle B \approx 40.6°}$

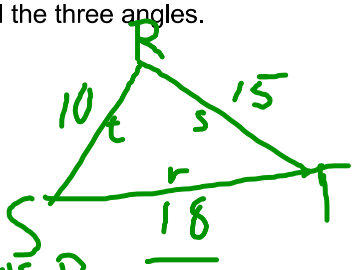
Find ∠C by subtraction

$180° - 85° - 40.6°$

$\boxed{\approx 54.4° = \angle C}$

Example 2 SSS:

Given Δ RST with sides r =18", s =15", and t =10". Find the three angles.



Draw ✓
Label ✓
Equation
Solve

$$18^2 = 10^2 + 15^2 - 2(10)(15)\underline{\cos R}$$

$$\frac{(18^2-10^2-15^2)}{(-2(10)(15))} = \cos R \approx .00\overline{3}$$

$$\cos^{-1}(\text{ans}) = \boxed{89.8}$$

Find ∠S: $\frac{\sin S}{15} = \frac{\sin 89.8}{18}$ $\boxed{\angle S = 56.4^\circ}$

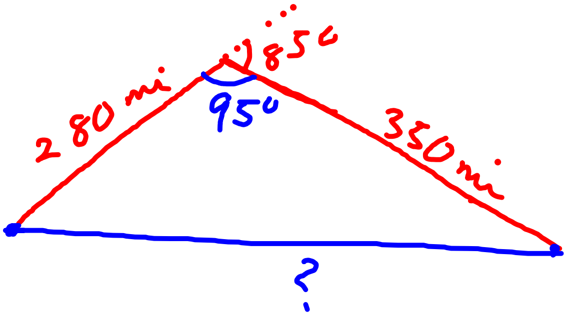
Find ∠T: angles add up to 180°.

so $\angle T = 180^\circ - 89.8^\circ - 56.4^\circ = \boxed{33.8^\circ}$

Example 3:

A plane flies 280 miles, turns 85º and flies another 350 miles. How far is it from the starting point?

Draw a picture.
Label it.
Determine which law to use.
Solve it.



$$d^2 = 280^2 + 350^2 - 2(280)(350)\cos 95^\circ$$

$$d^2 \approx 217,982.5256\ldots$$

$$d = \sqrt{ans} \approx \underline{\underline{466.89\,mi}}$$

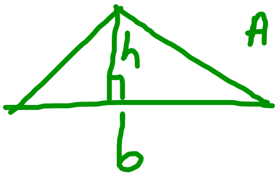
The area of a triangle in two ways:

✓ Area = 1/2 a$\overset{h}{\text{b sin}}$ C

or

Area = $\sqrt{s(s-a)(s-b)(s-c)}$ where s = semiperimeter, $\frac{a+b+c}{2}$

The second is called Heron's formula.

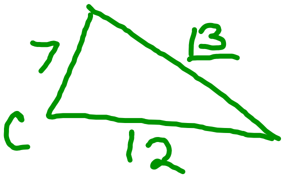


Area = $\frac{1}{2}bh$

Find the area of a triangle with sides 7cm, 12 cm, and 13 cm.

Use first formula:

Area = 1/2 ab sin C



$13^2 = 7^2 + 12^2 - 2(7)(12)\cos C$

$\frac{(13^2 - 7^2 - 12^2)}{(-2(7)(12))} = \cos C \approx .142857$

$\angle C = \cos^{-1}(\text{ans}) \approx 81.8^\circ$

$\frac{1}{2}(7)(12)\sin 81.8^\circ$

41.57 cm²

Use Heron's formula

$s = \frac{7+12+13}{2} = 16$

$\text{Area} = \sqrt{16(16-7)(16-12)(16-13)}$

$= \sqrt{16 \cdot 9 \cdot 4 \cdot 3}$

$= 4 \cdot 3 \cdot 2\sqrt{3}$

$= 24\sqrt{3}$

$\approx 41.57 \text{ cm}^2$