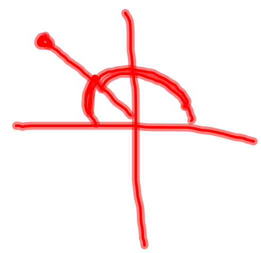
TRIG 3.1 ~ Law of Sines

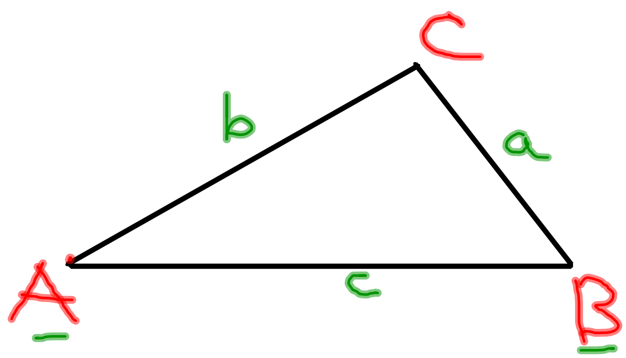
- Prove the Law of Sines
- Use the Law of Sines to solve triangles.
- Identify when the solution to the triangle is ambiguous.
- Find the area of triangles.

$\sin 120^\circ = \sin 60^\circ$



We will now apply our techniques to solving oblique triangles (those with no right angles.)

How to label sides and angles:

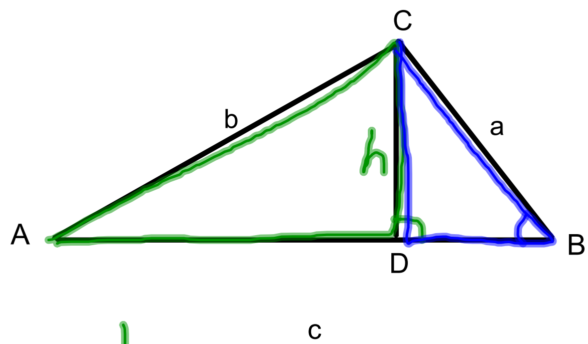


Law of Sines: If ABC is a triangle with sides a,b,c then

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

α β γ

Proof: Given triangle ABC
Draw altitude $\overline{CD}$ to side $\overline{AB}$
Let CD = h



In ΔADC, sin A = $\frac{h}{b}$

In ΔBCD, sin B = $\frac{h}{a}$

Solve each for h= $h = b\sin A$ h = $a\sin B$

$$b\sin A = a\sin B$$

$$\frac{b}{\sin B} = \frac{a}{\sin A} = \frac{c}{\sin C}$$

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

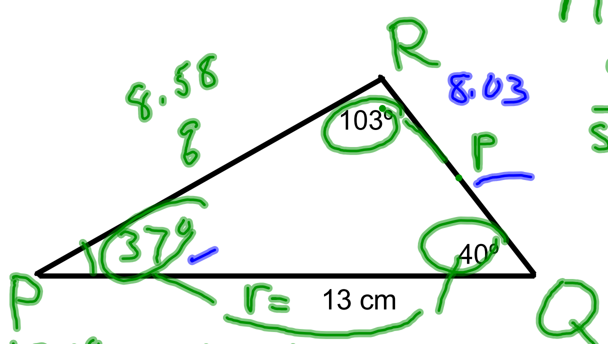
Example 1:

Solve for the missing sides and angle.

180° Sum of 3 ∠s

AAS

ASA



$180° - 103° - 40° =$

$$\frac{q}{\sin 40°} = \frac{13}{\sin 103°}$$

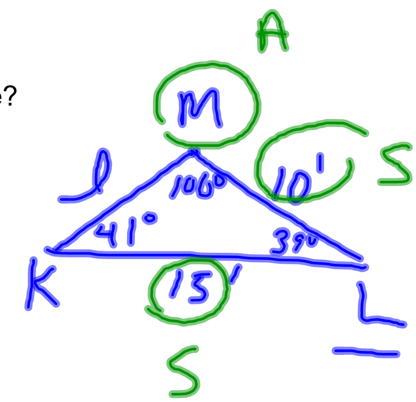
$$q = \frac{13 \sin 40°}{\sin 103°} \approx 8.58 \text{ cm}$$

$$\frac{p}{\sin 37°} = \frac{13}{\sin 103°} \approx 8.03 \text{ cm}$$

Example 2: What if we are looking for an angle?

Triangle MKL with $\angle M = 100°$
$m = 15'$ $k = 10'$

Solve for the remaining parts of the triangle.



$$\frac{\sin K}{10} = \frac{\sin 100°}{15'}$$

$$\sin K = \frac{10 \sin 100°}{15} \approx .6565385\ldots$$

$\sin^{-1}(\text{Ans})$

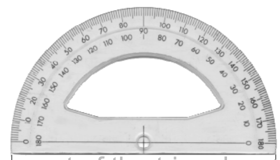
$$\angle K \approx 41°$$

$$\frac{l}{\sin 39°} = \frac{15}{\sin 100°}$$

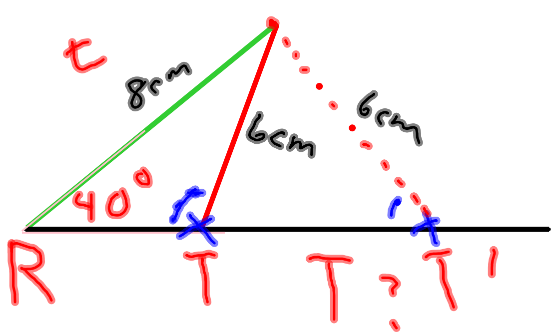
$$l = \frac{15 \sin 39°}{\sin 100°} \approx 9.59'$$

Example 3: The ambiguous case

Remember from Geometry the dreaded SSA?



Given triangle RST with ∠R = 40º, t = 8 cm and r = 6 cm, solve for the rest of the triangle.



$$\frac{\sin 40^\circ}{6} = \frac{\sin T}{8}$$

$$\sin T = \frac{8 \sin 40^\circ}{6} \approx .85705\ldots$$

$\angle T \approx 59^\circ$

or

$\angle T \approx 121^\circ$

$\angle T = 59^\circ \quad \angle S = 81^\circ$

$$\frac{6}{\sin 40^\circ} = \frac{s}{\sin 81^\circ}$$

$s \approx 9.22$ cm or

$\angle T = 121^\circ \quad \angle S = 180^\circ - 121^\circ - 40^\circ = 19^\circ$

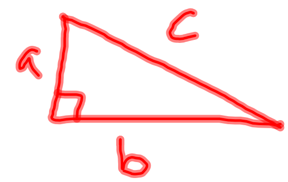
$$\frac{6}{\sin 40^\circ} = \frac{s}{\sin 19^\circ}$$

$s \approx 3.04$ cm

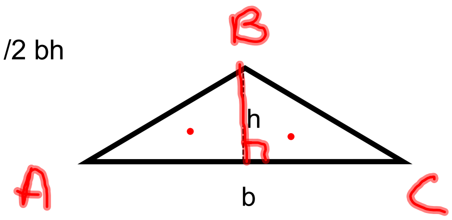
Two viable triangles.

Ambiguous!

Finding the area of a traingle.



Area of triangle = 1/2 bh



Area $= \frac{1}{2} ba$

$h = c \sin A$

$h = a \sin C$

Area $= \frac{1}{2} bc \sin A$ or Area $= \frac{1}{2} ba \sin C$

area of any triangle is $\frac{1}{2}$ the product of 2 sides and the sine of the angle between the two sides.

Area $= \frac{1}{2} ac \sin B$