1.7 ~ Inverse Trigonometric Functions

You will learn to:

Evaluate and graph the inverse sine function.
Evaluate and graph the other inverse trigonometric functions.

The inverse of a function *f(x)* is written $f^{-1}(x)$, pronounced f inverse of x.

The -1 is NOT an exponent.
The original function must be 1-to-1.
The inverse is a reflection through the line y = x
An (a,b) pair on the function becomes a (b,a) pair on the inverse.
The domain of *f(x)* is the range of $f^{-1}(x)$ and visa versa.

$\sin^2 x = (\sin x)^2$

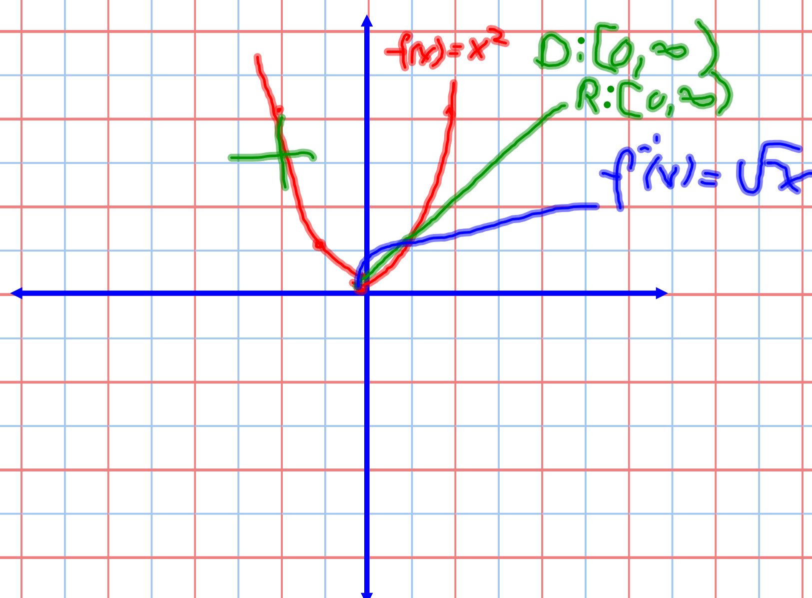
~~but $\sin^{-1} x \neq \frac{1}{\sin x}$~~

Example: Inverse of $y = x^2$

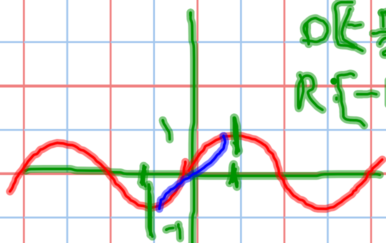
$f(x) = x^2$

$f^{-1}(x) = \sqrt{x}$

$\sqrt{16} = 4$



y = sin x



D$[-\frac{\pi}{2}, \frac{\pi}{2}]$

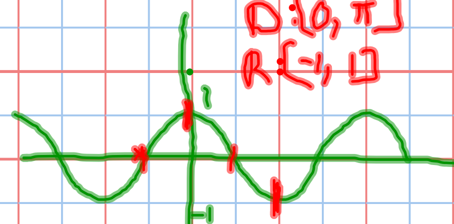
R:$[-1, 1]$

$f(x) = \sin^{-1}(x) = \arcsin x$

D: $[-1, 1]$

R: $[-\frac{\pi}{2}, \frac{\pi}{2}]$

y = cos x



D:$[0, \pi]$

R:$[-1, 1]$

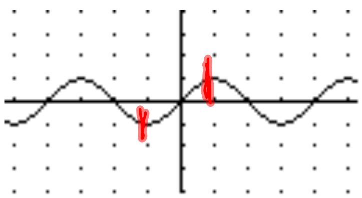
inverse

$y = \cos^{-1}(x) = \arccos x$

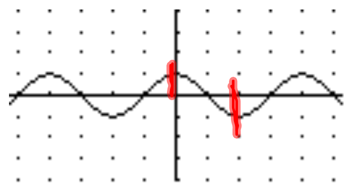
D:$[-1, 1]$

R:$[0, \pi]$

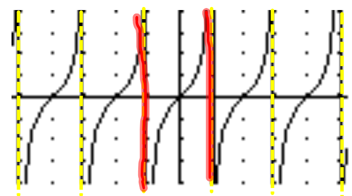
y = sin x



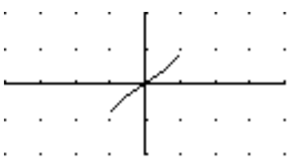
y = cos x



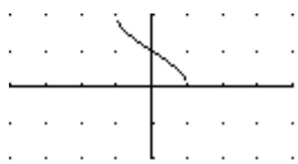
y = tan x



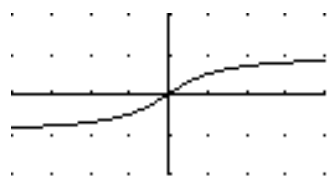
y = sin-1 x



y = cox-1 x



y = tan-1 x



The important thing to remember is the answer to a question about an inverse function is unique and must come from a certain range.

Arcsin x must have an answer in the interval $[-\pi/2, \pi/2]$.

As will the arccsc x, arctan x and arccot x functions.

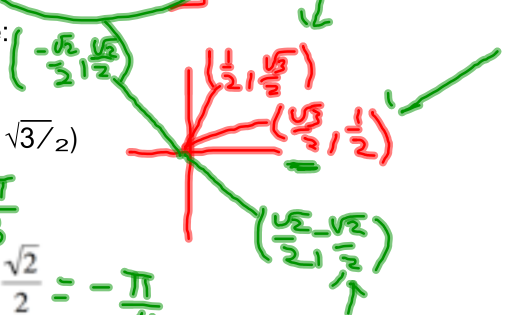
$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Arccos x must have an answer in the interval $[0, \pi]$.

As will the arcsec x function.

$[0\ \pi]$

Try these:

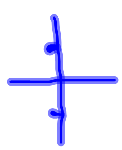


$\sin^{-1}(\sqrt{3}/2)$

$\frac{\pi}{3}$

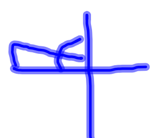
$\sin^{-1} -\frac{\sqrt{2}}{2} = -\frac{\pi}{4}$

$\cos^{-1} 0 = \frac{\pi}{2}$

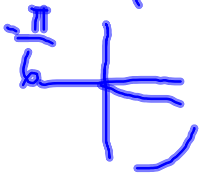


$\sec^{-1}(-2/\sqrt{3}) = \frac{5\pi}{6}$

$\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{5\pi}{6}$



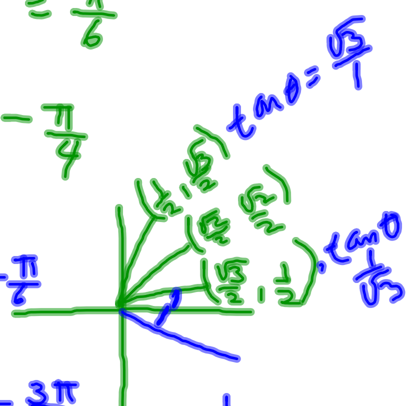
$\tan^{-1}(-1/\sqrt{3}) = -\frac{\pi}{6}$



$\sec^{-1} -1 = \pi$

$\cos^{-1}(\sqrt{3}/2) = \frac{\pi}{6}$

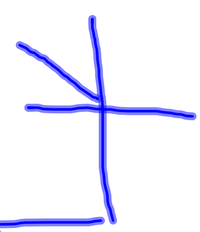
$\tan^{-1} -1 = -\frac{\pi}{4}$



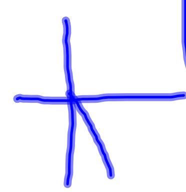
$\sin^{-1}(-1/2) = -\frac{\pi}{6}$

$\sec^{-1} -\sqrt{2} = \frac{3\pi}{4}$

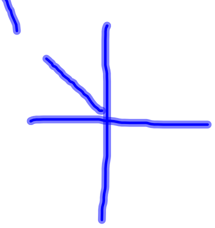
$\cos^{-1} = -\frac{1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$



$\tan^{-1} -\sqrt{3} = -\frac{\pi}{3}$



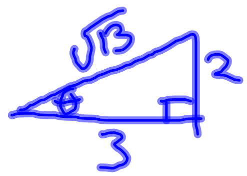
$\cos^{-1} -\frac{\sqrt{2}}{2} = \frac{3\pi}{4}$



Some more complex problem involving arcsin, arccos and arctan:
Hint: Draw a right triangle!

a) cos (arctan (2/3))

$= \frac{3}{\sqrt{13}}$



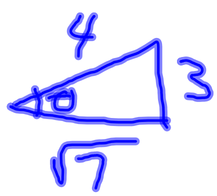
$2^2 + 3^2 = h^2$

$4 + 9 = h^2$

$\sqrt{13} = h$

b) tan (sin^{-1}(3/4))

$= \frac{3}{\sqrt{7}}$



$4^2 - 3^2 = s^2$

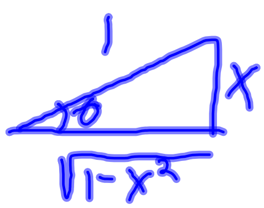
$16 - 9 = s^2$

$\sqrt{7} = s$

θ

c) sec (arcsin x)

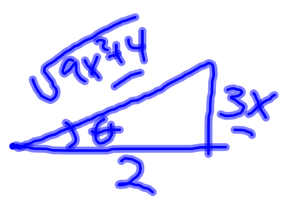
$= \frac{1}{\sqrt{1-x^2}}$



θ

d) csc (tan^{-1}(3x/2))

$= \frac{\sqrt{9x^2+4}}{3x}$



$h^2 = 9x^2 + 4$

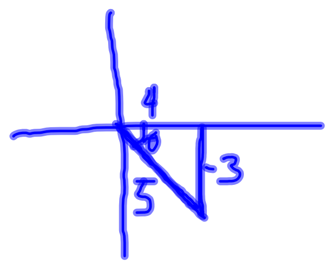
$h = \sqrt{9x^2+4}$

And a few more:

θ

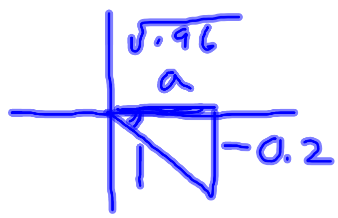
a) sec (arctan(-3/4))

$\frac{5}{4}$



b) cot(sin⁻¹(-0.2))

$\frac{\sqrt{.96}}{-.2}$

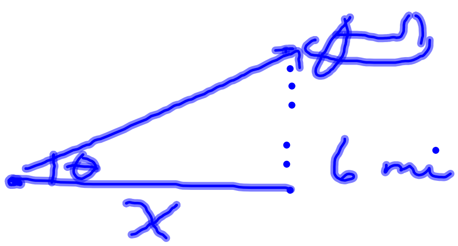


$a = \sqrt{1^2 - (.2)^2}$

$a = \sqrt{1 - .04}$

$a = \sqrt{.96}$

c) A plane flies at an altitude of 6 miles toward a point directly over an observer. Write the angle ø as a function of x, the horizontal distance from the observer to a point on the ground directly below the airplane.



$\tan\theta = \frac{6}{x}$

$\tan^{-1}\frac{6}{x} = \theta$

$$\sec^{-1}(2.3)$$

$$\cos^{-1}\left(\frac{1}{2.3}\right) \approx 1.12$$