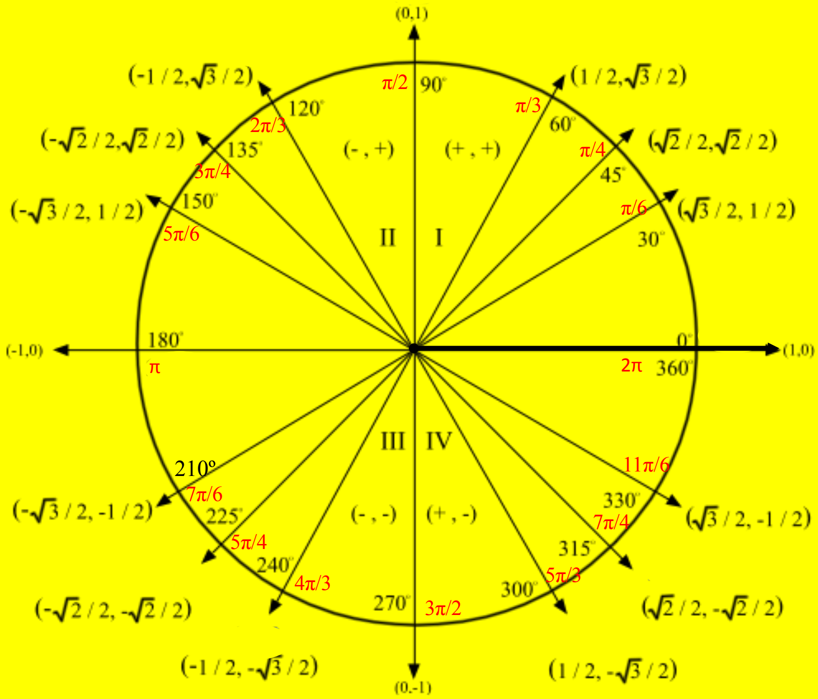
# Trig 1.2 part 2 ~ The Trigonometric Functions

You will
* Evaluate trigonometric functions using the unit circle.
* Use the domain and period to evaluate sine and cosine functions.
* Identify the reference angle of any angle on the unit circle.
* Use a calculator to evaluate trigonometric functions.

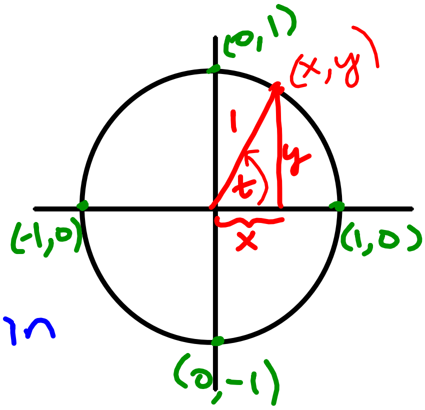
The Unit Circle



# Trigonometric functions

(on the unit circle)

unit circle = circle centered at origin w/ a "unit" radius, i.e. $r = 1$.




$t$ = angle measured from pos. x-axis in counter-clockwise direction

Sine $t$ = $y$

Cosine $t$ = $x$

Cosecant $t$ = $\frac{1}{y}$

Secant $t$ = $\frac{1}{x}$

Tangent $t$ = $\frac{y}{x}$

Cotangent $t$ = $\frac{x}{y}$

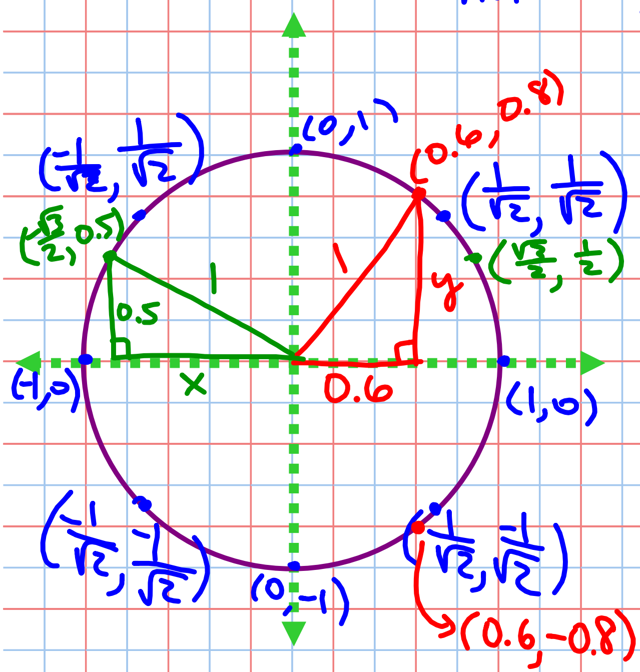
Note: all of the above definitions are only true when $(x,y)$ is the pt on the unit circle that we get to by travelling the angle $t$.

# Other points on the unit circle:
# radius = 1.

Equation: $x^2+y^2=1$

$\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ $\left(\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ $\left(\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$ $\left(\frac{-1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$

note: $\frac{1}{2}+\frac{1}{2}=1 \Rightarrow x=\pm\frac{1}{\sqrt{2}}$, $y=\pm\frac{1}{\sqrt{2}}$




Complete these ordered pairs:

a. (0.6, ?)

$0.6^2+y^2=1^2 \Rightarrow y^2=0.64$

$y=\pm 0.8$

b. ( ? , 0.5)

$0.5^2+x^2=1^2 \Rightarrow x^2=0.75=\frac{3}{4}$

$x=\frac{\pm\sqrt{3}}{2}$

c. (-0.8, ?)

$(-0.8)^2+y^2=1^2 \Rightarrow y=\pm 0.6$

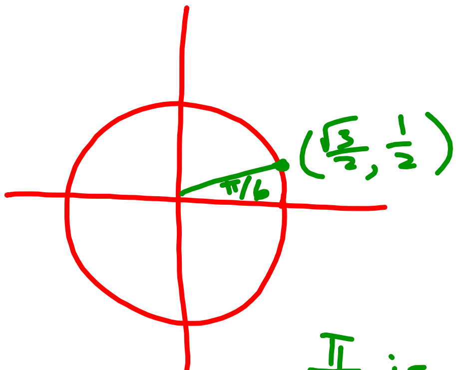
d. ( ?, 0.1)

$x^2+0.1^2=1^2$

$x^2=0.99$

$x=\pm 0.3\sqrt{11}$

# Periodic means what??




$\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$

$\sin\frac{\pi}{6} = \frac{1}{2}$

$\frac{\pi}{6}$ is coterminal with $\frac{\pi}{6} + 2n\pi$, $n \in \mathbb{Z}$

(n is any integer)

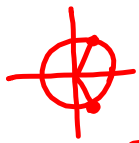
$\sin\left(\frac{\pi}{6} + 2n\pi\right) = \sin\left(\frac{\pi}{6}\right)$

and $\cos\left(\frac{\pi}{6} + 2n\pi\right) = \cos\left(\frac{\pi}{6}\right)$

$\Rightarrow$ sine & cosine are periodic fns because they repeat, i.e. they output same values repetitively (in an oscillatory fashion)

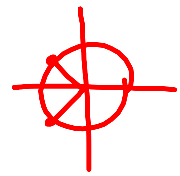
In fact, all trig fns are periodic.

Back to the unit circle -- Answer each of these and come up with a conjecture.



$\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$

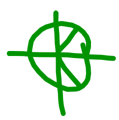
$\sin\left(-\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2}$



$\sin\left(\frac{3\pi}{4}\right) = \frac{\sqrt{2}}{2}$

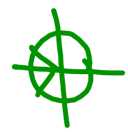
$\sin\left(-\frac{3\pi}{4}\right) = -\frac{\sqrt{2}}{2}$

Conjecture: $\sin(-\theta) = -\sin\theta$



$\cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$

$\cos\left(-\frac{\pi}{3}\right) = \frac{1}{2}$



$\cos\left(\frac{3\pi}{4}\right) = -\frac{\sqrt{2}}{2}$

$\cos\left(-\frac{3\pi}{4}\right) = -\frac{\sqrt{2}}{2}$

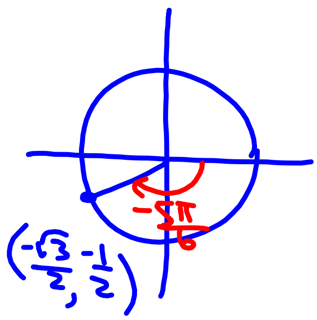
conjecture: $\cos(-\theta) = \cos(\theta)$

Facts:

$\boxed{\begin{array}{l} \sin(\theta) \text{ is odd fn.} \\ \cos(\theta) \text{ is even fn.} \end{array}}$

Exercise 1:

a. Evaluate the six trigonometric functions of *t* if $t = -5\pi/6$.




$\sin t = \frac{-1}{2}$

$\cos t = \frac{-\sqrt{3}}{2}$

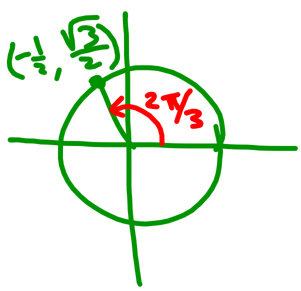
$\tan t = \frac{-1/2}{-\sqrt{3}/2} = \frac{1}{\sqrt{3}}$ or $\frac{\sqrt{3}}{3}$

$\csc t = -2$

$\sec t = \frac{-2}{\sqrt{3}}$ or $\frac{-2\sqrt{3}}{3}$

$\cot t = \sqrt{3}$

b. Evaluate the six trigonometric functions of *t* if $t = 2\pi/3$.




$\cos t = \frac{-1}{2}$

$\sin t = \frac{\sqrt{3}}{2}$

$\tan t = \frac{\sqrt{3}/2}{-1/2} = -\sqrt{3}$

$\sec t = -2$

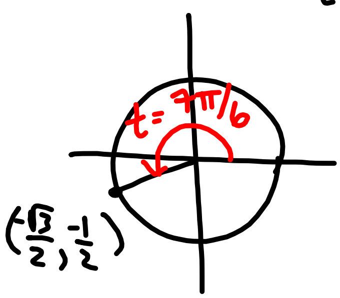
$\csc t = \frac{2}{\sqrt{3}}$ or $\frac{2\sqrt{3}}{3}$

$\cot t = \frac{-1}{\sqrt{3}}$ or $\frac{-\sqrt{3}}{3}$

Practice these:

Exercise 2:

If $sin\ t = -0.5$ and $\underbrace{\pi < t < 3\pi/2}_{t \text{ in } Q3}$, determine the other five trigonometric functions of $t$.




$t = \frac{7\pi}{6}$

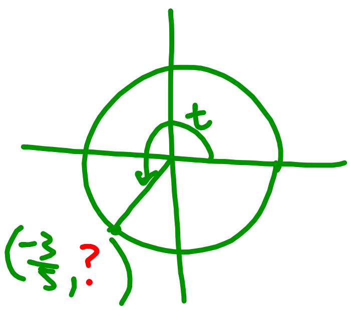
$\sin t = -\frac{1}{2}$ $\quad$ $\csc t = -2$

$\cos t = -\frac{\sqrt{3}}{2}$ $\quad$ $\sec t = \frac{-2}{\sqrt{3}}$

$\tan t = \frac{-1/2}{-\sqrt{3}/2} = \frac{1}{\sqrt{3}}$ $\quad$ $\cot t = \sqrt{3}$

Exercise 3:

If $sec\ t = -5/3$ and $t$ is in the third quadrant, determine the other five trigonometric functions of $t$.




$\sec t = \frac{-5}{3} \Rightarrow \cos t = \frac{-3}{5}$

Use Pythagorean Identity:

$$\sin^2\theta + \cos^2\theta = 1$$

$$\sin^2 t + \left(\frac{-3}{5}\right)^2 = 1$$

$$(\sin t)^2 + \frac{9}{25} = 1$$

$$(\sin t)^2 = \frac{16}{25}$$

$$\sin t = \pm\sqrt{\frac{16}{25}} = \frac{\pm 4}{5}$$

in Q3, sin t is negative.

$\Rightarrow \sin t = \frac{-4}{5}$

$\cos t = \frac{-3}{5}$

$\sin t = \frac{-4}{5}$

$\tan t = \frac{-4/5}{-3/5} = \frac{4}{3}$

$\sec t = \frac{-5}{3}$

$\csc t = \frac{-5}{4}$

$\cot t = \frac{3}{4}$

One more thing - A <u>reference angle</u> is (notated as $\theta'$)

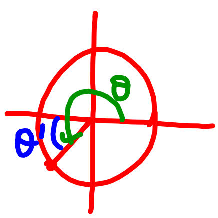
Positive
Acute
Shares the terminal side with the original angle and has one side on the $x$-axis.
If the angle is in radians, the reference angle is in radians.
If the angle is in degrees, the reference angle is in degrees.

Every angle $\theta$ has a reference angle $\theta'$.

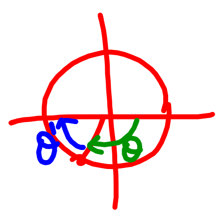
Note: The quadrant angles have no reference angle.

Examples:

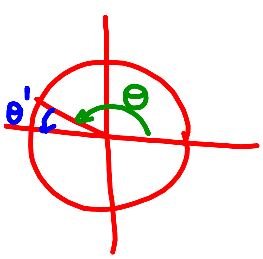
$\theta = 5\pi/4 \Rightarrow \theta' = \frac{\pi}{4}$



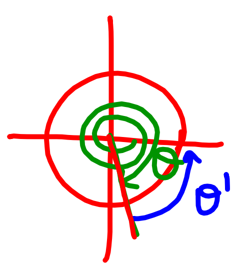

$\theta = -2\pi/3 \Rightarrow \theta' = \frac{\pi}{3}$




$\theta = 140^o \Rightarrow \theta' = 40^\circ$




$\theta = -800^o \Rightarrow \theta' = 80^\circ$



Exercise 4:

Of course a calculator will provide approximate answers in decimal form and approximate answers for "unfriendly" angles.

(to 4 decimal places)

$\sin(3\pi/4) = \frac{\sqrt{2}}{2} \simeq 0.7071$

$\tan(5\pi/6) = \frac{-1}{\sqrt{3}} \simeq -0.5774$

$\cos(2\pi/3) = \frac{-1}{2} = -0.5$

$\sec(5\pi/4) = -\sqrt{2} \simeq 1.4142$

$\sin(0.24) \simeq 0.2377$