Homework 9, Math 6610, due Nov. 7

Consider the matrix

$$A = \begin{bmatrix} -261 & 209 & -49 \\ -530 & 422 & -98 \\ -800 & 631 & -144 \end{bmatrix},$$

where $\lambda(A) = \{10, 4, 3\}$.

In each of the following, determine the rate of convergence for the eigenvalue computation by computing e_{k+1}/e_k for large k values, where e_k is the error at the k-th iteration.

1. Write a power method program to compute the largest eigenvalue 10, starting from three different initial guesses:

$$x_o = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

2. Write an inverse iteration program, starting from $\sigma = 3.51$ and 3.49. Choose whatever initial guess you prefer.

3. Implement the orthogonal iteration algorithm. Get all three eigenvalues from the resulting $Q^T A Q$.

4. Implement the original QR iteration algorithm. Get all three eigenvalues from the last QR.