Homework 7, Math 6610-1, Due Oct. 24

1. Let A be $m \times n$, $m < n$, and a linear system $Ax = b$, $b \in R^m$, to solve for $x \in R^n$. This linear system has no solution or has infinitely many solutions. Describe the condition that distinguishes these two cases. In the case with infinitely many solutions, show that the solution is an $(n - m)$-dimensional set. In this case we would be interested in the minimal 2-norm solution. Show how to compute this unique solution using QR, and SVD approaches.

2. The least square minimization over a sphere is an important problem. Namely, we want to minimize $||Ax - b||_2$, subject to the constraint $||x||_2 \leq \alpha$, with $\alpha > 0$. Here $A \in R^{m \times n}$, $b \in R^m$, and A is assumed to have full rank. Use SVD to derive the solutions for different cases resulting from the relationships among A, b and α.