Homework 4, Math 6610-1, Due Oct. 3

Consider the tridiagonal matrix

$$A = \begin{bmatrix} a_1 & -b_1 & & & & \\ -b_1 & a_2 & -b_2 & & & \\ & \cdot & \cdot & \cdot & & \\ & & \cdot & \cdot & \cdot & \\ & & & -b_{n-2} & a_{n-1} & -b_{n-1} \\ & & & & -b_{n-1} & a_n \end{bmatrix},$$

where $b_0, b_1, \ldots, b_{n-1}, b_n$ are random numbers drawn from a log-normal distribution, that is, $\log b_i \sim N(\mu, \sigma^2)$, and

$$a_i = b_{i-1} + b_i, \quad i = 1, \ldots, n.$$

1. Develop an algorithm of Gaussian elimination without pivot for such matrices to solve the linear system $A\mathbf{x} = \mathbf{b}$, and write a Matlab code. Let $n = 200$ and avoid using 2-dimensional arrays in the program.

2. Fix the parameters $\mu = 0, \sigma = 1$ and choose some random $\mathbf{x}$ (with each component of $\mathbf{x}$ drawn from the standard normal distribution), calculate $\mathbf{b}$ for each case by multipliication, then solve the system for $\mathbf{b}$ so you will have the exact error $\delta\mathbf{x}$. Compare the error bounds (2.13) and (2.14) on page 54 of the textbook with the actual error $||\mathbf{x} - \hat{\mathbf{x}}||_\infty$. Present you results in a form similar to figure 2.3 on page 56. You should generate 20 different vectors $\mathbf{b}$ for comparisons.

3. Change the parameters μ to -10 and 10, and σ to 0.2 and 2 (with all combinations), repeat step 2 and discuss what you observe. Do you think we need pivoting for this matrix?