Homework 3, Math 6610-1, Due Sept. 26

1. Consider the Gaussian elimination with partial pivoting for a nonsingular matrix A as described in algorithm 2.2. By identifying the matrices L, U and P, we can confirm the LU decomposition of A in the form $PA = LU$. The actual steps take the form
$$L_{n-1}P_{n-1}\cdots L_3P_3L_2P_2L_1P_1A = U,$$
as in each step, a row permutation is performed (interchanging two rows between row i and row k where $k > i$) and then the eliminations are executed. Write the matrices L_i and P_i explicitly and verify that the product in the above equation can be written as
$$L'_{n-1}\cdots L'_3L'_2L'_1P_{n-1}\cdots P_3P_2P_1A,$$
where
$$L'_{n-1} = L_{n-1}, \quad L'_{n-2} = P_{n-1}L_{n-2}P_{n-1}^{-1}, \quad \cdots, \quad L'_1 = P_{n-1}\cdots P_2L_1P_2^{-1}\cdots P_{n-1}^{-1}.$$
Show that L'_i and L_i have the same structure with the only difference in that some subdiagonal entries are reordered. Therefore, we can write $L'PA = U$, where $L' = L'_{n-1}\cdots L'_3L'_2L'_1$, which is equivalent to $PA = LU$, where $L = (L')^{-1} = (L'_1)^{-1}(L'_2)^{-1}\cdots(L'_{n-1})^{-1}$ is the lower triangular matrix in the LU decomposition. To illustrate the idea, you can work on a 4×4 matrix and generalize your conclusion to general matrices.

2. Question 2.2 on page 94 in Demmel.

3. Question 2.17 on page 98 in Demmel.

4. Gaussian elimination can be used to compute the inverse A^{-1} of the nonsingular $n \times n$ matrix A, though you should avoid doing so unless it's necessary. Describe an algorithm for computing A^{-1} by solving n systems of equations, and show that its asymptotic operation count is $8n^3/3$ flops.