Homework No. 1, Math 6610-1, Due Sept. 6

1. Prove the following lemmas:

   (a) For vector norms,

$$
\begin{aligned}
||x||_2 &\le ||x||_1 \le \sqrt{n}||x||_2, \\
||x||_\infty &\le ||x||_2 \le \sqrt{n}||x||_\infty, \\
||x||_\infty &\le ||x||_1 \le n||x||_\infty.
\end{aligned}
$$

   (b) An operator norm is a matrix norm.

   (c) For matrix norms,

      i. $||A||_\infty = \max_{x\neq 0} \frac{||Ax||_\infty}{||x||_\infty} = \max_i \sum_j |a_{ij}| =$ maximum absolute row sum.

      ii. $||A||_1 = \max_{x\neq 0} \frac{||Ax||_1}{||x||_1} = ||A^T||_\infty$.

      iii. $||A||_2 = \max_{x\neq 0} \frac{||Ax||_2}{||x||_2} = \sqrt{\lambda_{\max}(A^*A)}$, where $\lambda_{\max}$ denotes the largest eigenvalue.

      iv. If A is n-by-n, then

$$
n^{-1/2}||A||_2 \le ||A||_\infty \le n^{1/2}||A||_2
$$

2. Design a method to find the machine epsilon of your system, in both single and double precisions.