Math 5760/6890
Introduction to Mathematical Finance

Instructor: Jingyi Zhu
Office: LCB 335
Telephone:581-3236
E-mail: zhu@math.utah.edu
Class web page:
www.math.utah.edu/~zhu/5760_11f.html

What you should NOT expect to learn here:

- Predict stock movements
- Pick stocks to outperform the market
- Forecast a particular sector/market
- Predict a crash
- Anything about the future

What you hope to and will learn (if you make the efforts)

- Consolidate and extend your knowledge of time value of money (interest rates)
- Find financial instruments to hedge your stocks
- Price a call or a put option - using Black-Scholes formula
- Understand what's about optimal portfolios - for a particular consideration

Lecture 1: Basic Probability Notions

- Probability triple: sample space, events and probability
- Conditional probability: $P(A|B) = \dfrac{P(A \cap B)}{P(B)}$
- Independence: $P(A \cap B) = P(A) \cdot P(B)$
- Random variables, expectation $E[X] = \sum xP(X = x)$
- Jointly distributed RV's, covariance and correlation

$$Cov(X, Y) = E\left[(X - E[X]) \cdot (Y - E[Y])\right]$$

$$\rho(X, Y) = Corr(X, Y) = \frac{Cov(X, Y)}{\sqrt{Var(X) \cdot Var(Y)}}$$

Basic Probability Notions (continued)

- Conditional expectation

$$E[X|Y] = E[X|Y = y] = \sum_x xP(X = x|Y = y)$$

- A trivial but important observation

$$E[X] = E\left[E[X|Y]\right]$$

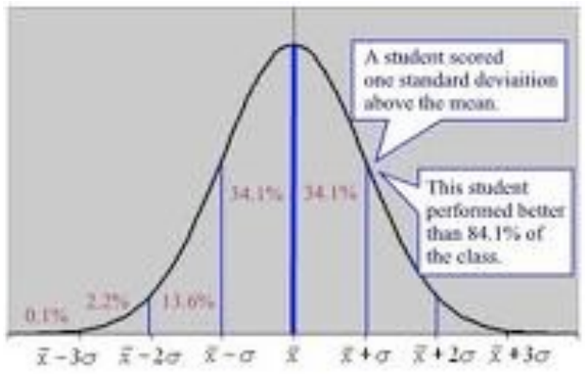
- Continuous random variable

$$P(a \leq X \leq b)$$

Normal Random Variables

- Bell-shaped density function

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$



- Center

- Spread - variance

- Adding the variances for two independent normal rv's

- Lognormal distribution $Y = e^X$

$$E[Y] = e^{\mu+\frac{1}{2}\sigma^2} \qquad Var(Y) = e^{2\mu+2\sigma^2} - e^{2\mu+\sigma^2}$$

Central Limit Theorem

- Most important theorem in probability theory
- Begin with any distribution (with finite mean and variance)
- A natural introduction to normal rv's
- Sum of iid (independent, identically distributed) rv's
- Properly scaled (square root of n)
- Converge in distribution

Investment Securities

- Equity: stocks
- Fixed-Income: bonds and papers
- Government vs corporations
- Rating and rating agencies (S&P, Moody and Fitch)
- Returns, interest rates

Financial Derivatives

- Securities to be traded, on exchange or over-the-counter
- Value derived from other securities, of other uncertain quantities that will be determined in the future
- “Written on the underlying”
- A call option example
  - set an expiration date, exercise price
  - payoff dependent on S-K: S-K, if positive, and zero if zero or negative

Other options on a stock

- A put option
- Invest in call if you expect the stock to go up and look for the most effective way to benefit, or invest in put otherwise
- European vs American:
  - European: you can exercise only on the expiration date
  - American: you can exercise any time before the expiration date
  - Expiration date: 3rd Friday of the month

Main Question in Option Pricing

- With an option contract, K (exercise or strike price) and T (expiration date) set
- How much is the worth of the contract TODAY?
- Depends on today's stock price
- Black-Scholes formula to compute
- Crucial parameter in the formula: the volatility

Modeling of the stock price

- Focus on the return over the time period

$$\frac{S(t+\Delta t)-S(t)}{S(t)}$$

- Collection of returns modeled as realizations for a rv

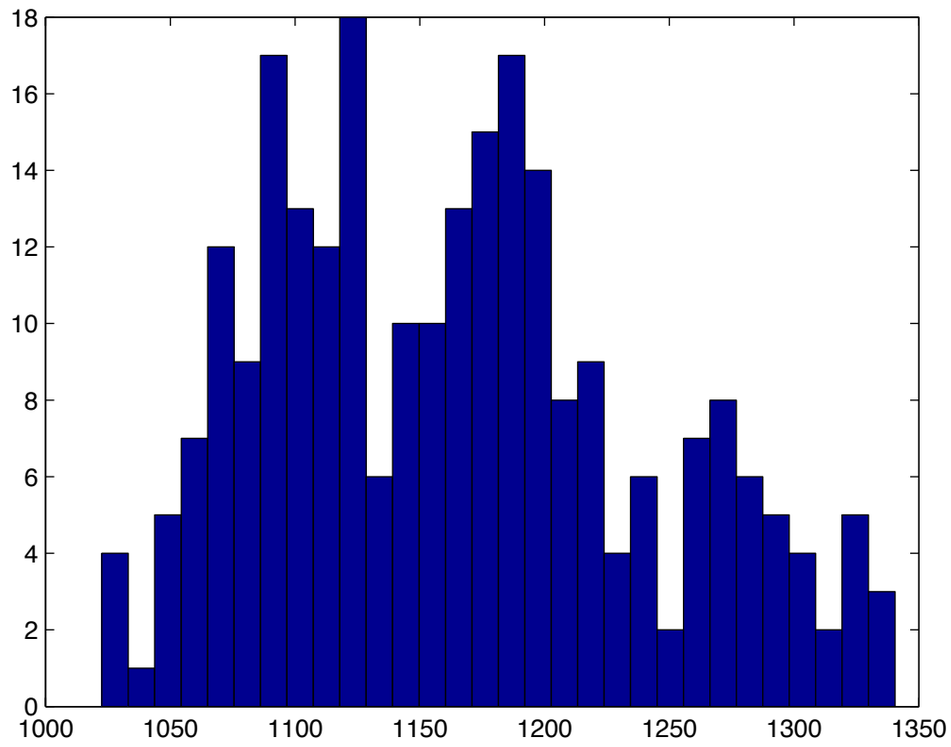
- Suggestion for the distribution?

- Normal distribution a natural choice

- Result in lognormal distribution for the stock itself

- Normal distribution for S not realistic!

S&P 500 Price Distribution



S&P 500 Daily Return Distribution