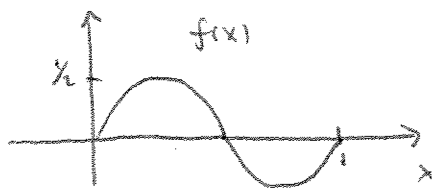
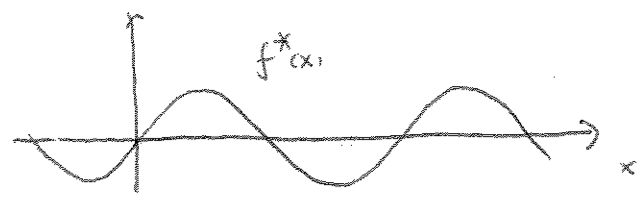
Section 3.4

2. $f(x) = \sin\pi x \cos\pi x = \frac{1}{2}\sin 2\pi x \qquad 0 < x < 1$

$f^*(x) = \frac{1}{2}\sin 2\pi x \qquad -\infty < x < \infty$



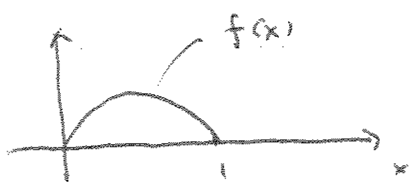


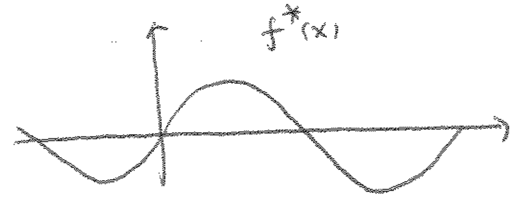
$$u(x,t) = \frac{1}{2}\left[\frac{1}{2}\sin 2\pi\left(x - \frac{1}{\pi}t\right) + \frac{1}{2}\sin 2\pi\left(x + \frac{1}{\pi}t\right)\right]$$

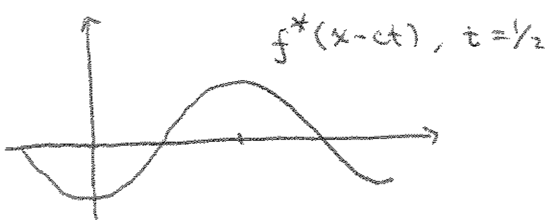
$$= \frac{1}{4}\left[\sin(2\pi x - 2t) + \sin(2\pi x + 2t)\right]$$

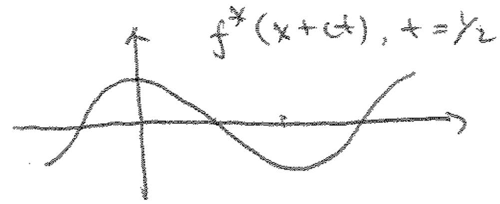
$$= \frac{1}{2}\sin 2\pi x \cos 2t$$

10.









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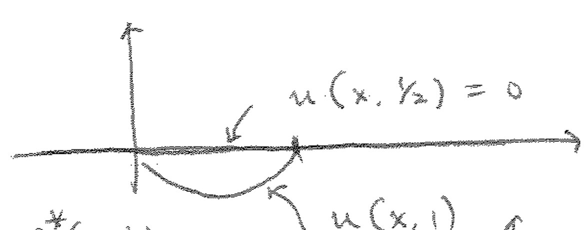

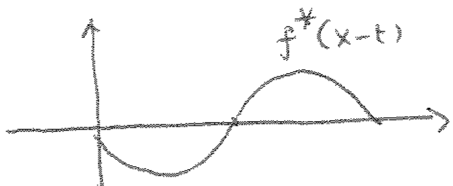

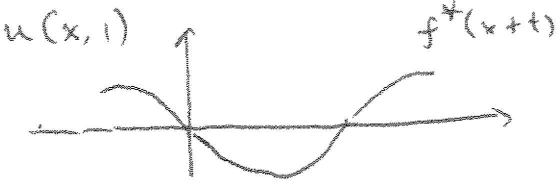

14. (a) $$u(0,t) = \frac{1}{2}\left[f^*(-ct) + f^*(ct)\right] + \frac{1}{2c}\int_{-ct}^{ct} g^*(s)\, ds$$

$$= 0$$

since f^* is odd, and

$$\int_{-ct}^{ct} g^*(s)\, ds = \int_{-ct}^{0} g^*(s)\, ds + \int_{0}^{ct} g^*(s)\, ds$$

change of variable $$= -\int_{ct}^{0} g^*(-z)\, dz + \int_{0}^{ct} g^*(s)\, ds$$

g^* is odd $$= \int_{ct}^{0} g^*(z)\, dz + \int_{0}^{ct} g^*(s)\, ds$$

$$= -\int_{0}^{ct} g^*(z)\, dz + \int_{0}^{ct} g^*(s)\, ds = 0$$

(c) $$u(L,t) = \frac{1}{2}\left[f^*(L-ct) + f^*(L+ct)\right] + \frac{1}{2c}\int_{L-ct}^{L+ct} g^*(s)\, ds$$

$$= \frac{1}{2}\left[f^*(L-ct) + f^*(L+ct-2L)\right] + \frac{1}{2c}\int_{-ct}^{ct} g^*(z+L)\, dz$$

Notice $\tilde{g}(z) = g^*(z+L)$ is also odd, since

$$\hat{g}(-z) = g^*(-z+L) \underset{\text{periodicity}}{=} g^*(-z-L) = g^*(-(z+L))$$

$$= -g^*(z+L) = -\hat{g}(z)$$

then $$\int_{-ct}^{ct} g^*(z+L)\, dz = \int_{-ct}^{ct} \tilde{g}(z)\, dz = 0$$ as we proved.

(b) $$u(x,0) = \frac{1}{2}\left[f^*(x) + f^*(x)\right] + \frac{1}{2c}\int_{x}^{x} g^*(s)\, ds = f^*(x)$$

$$= f(x)$$ since $0 < x < L$.