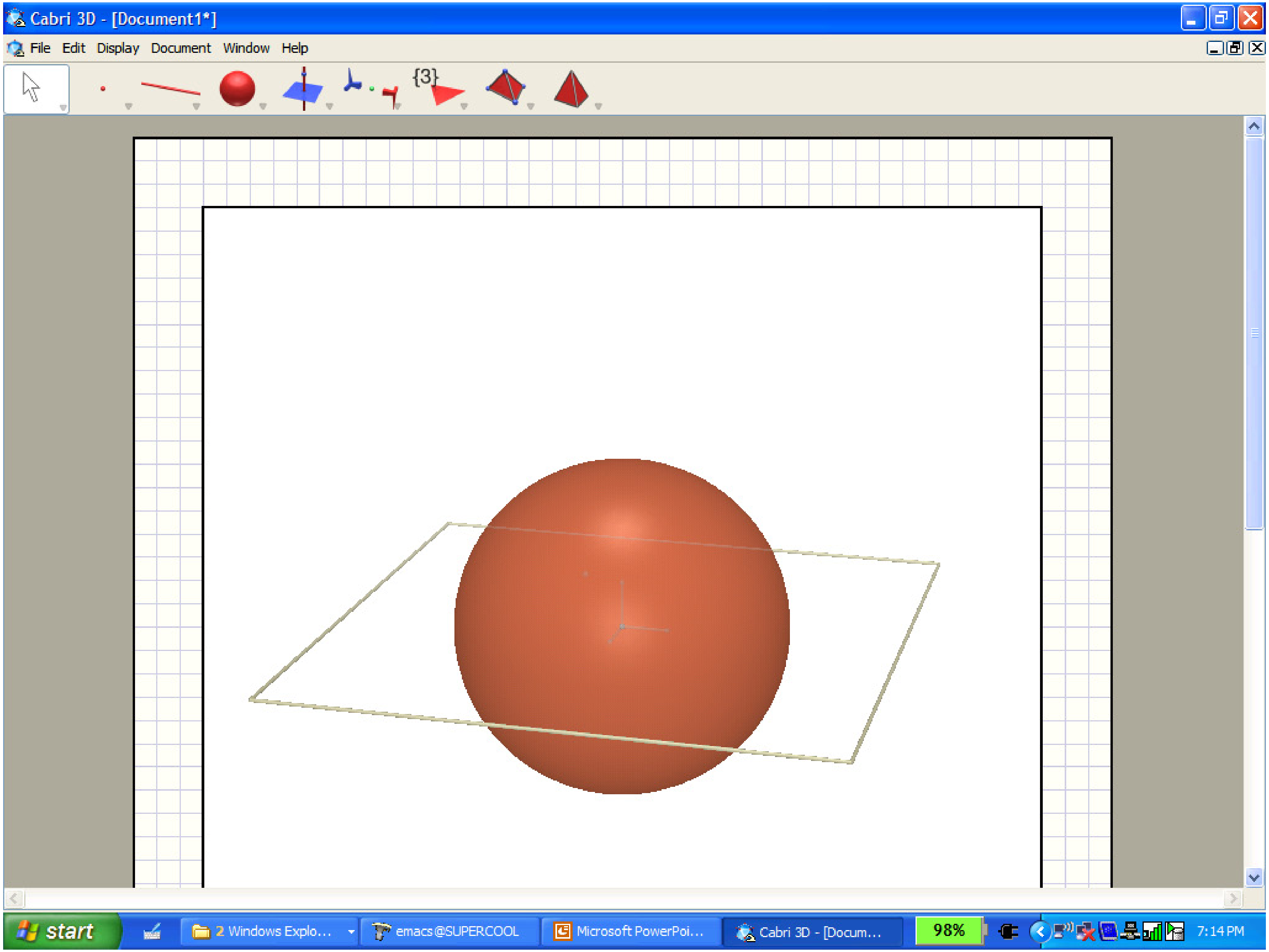
# Class #9

Projective plane, affine plane,
hyperbolic plane,

# Interpretation #5

- Points are points on a sphere
  $S^3=\{(x, y, z): x^2 +y^2 +z^2 =1\}$
- Lines are great circles (great circles are circles of unit radius with the center at the origin)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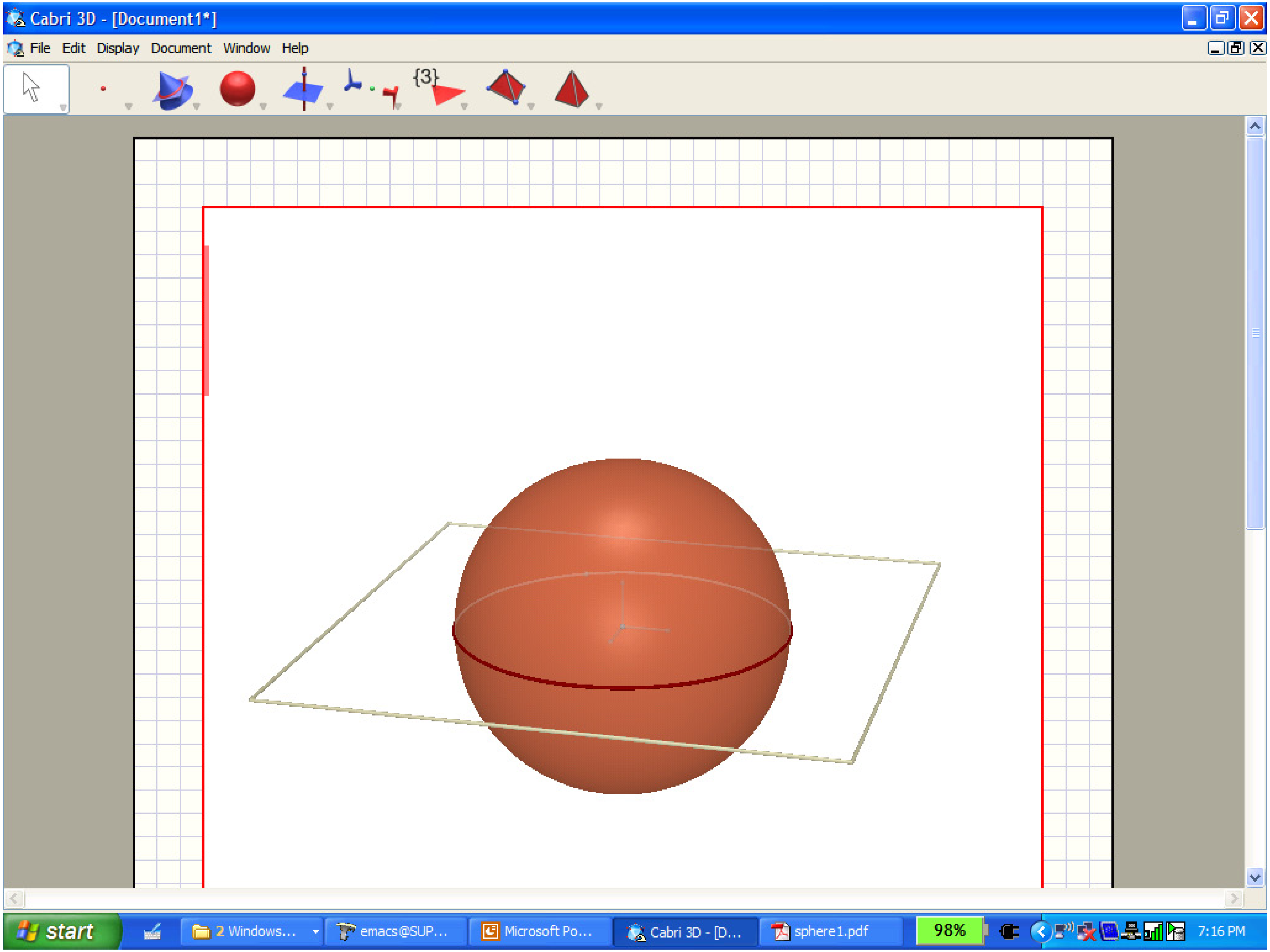
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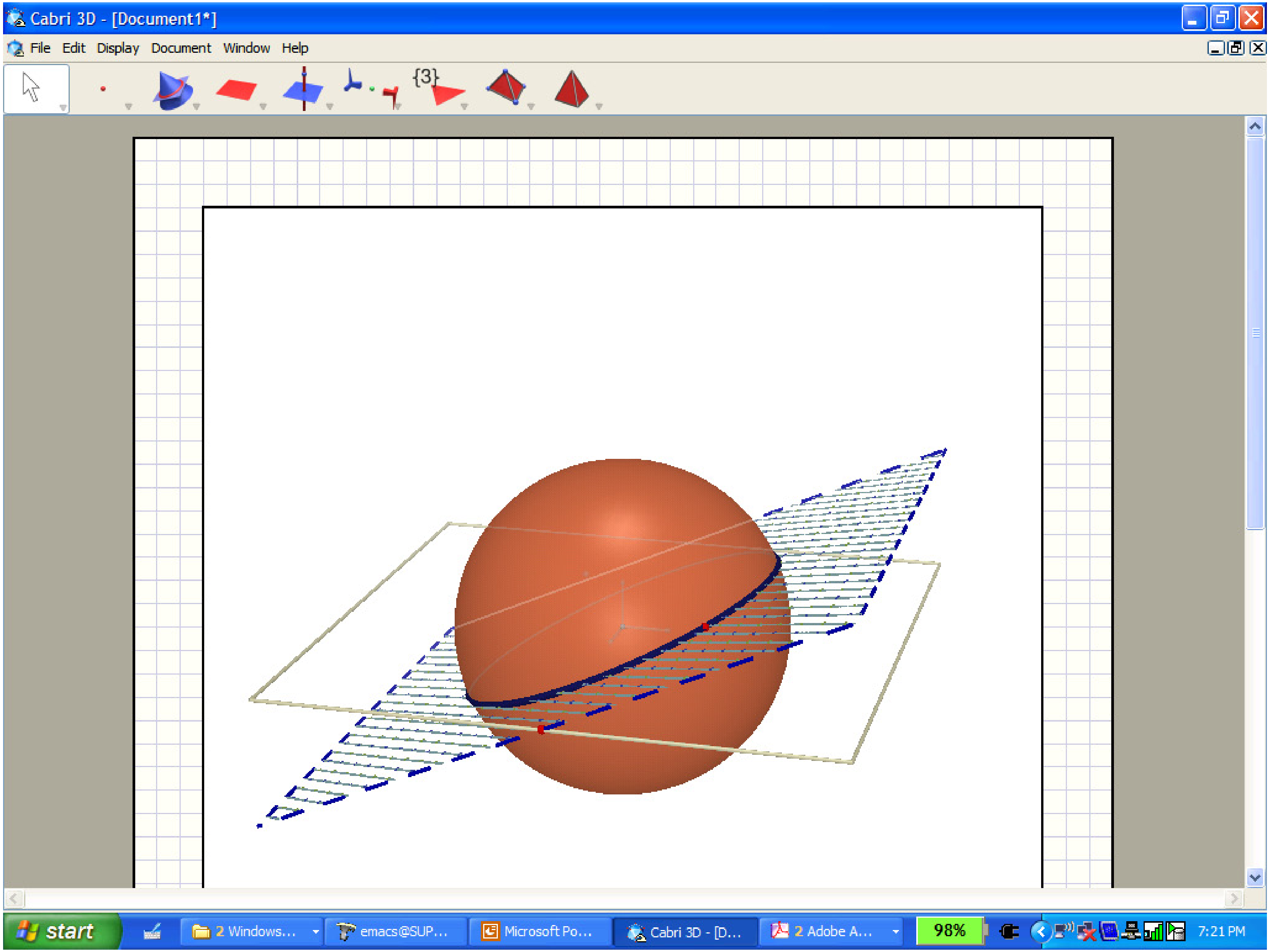
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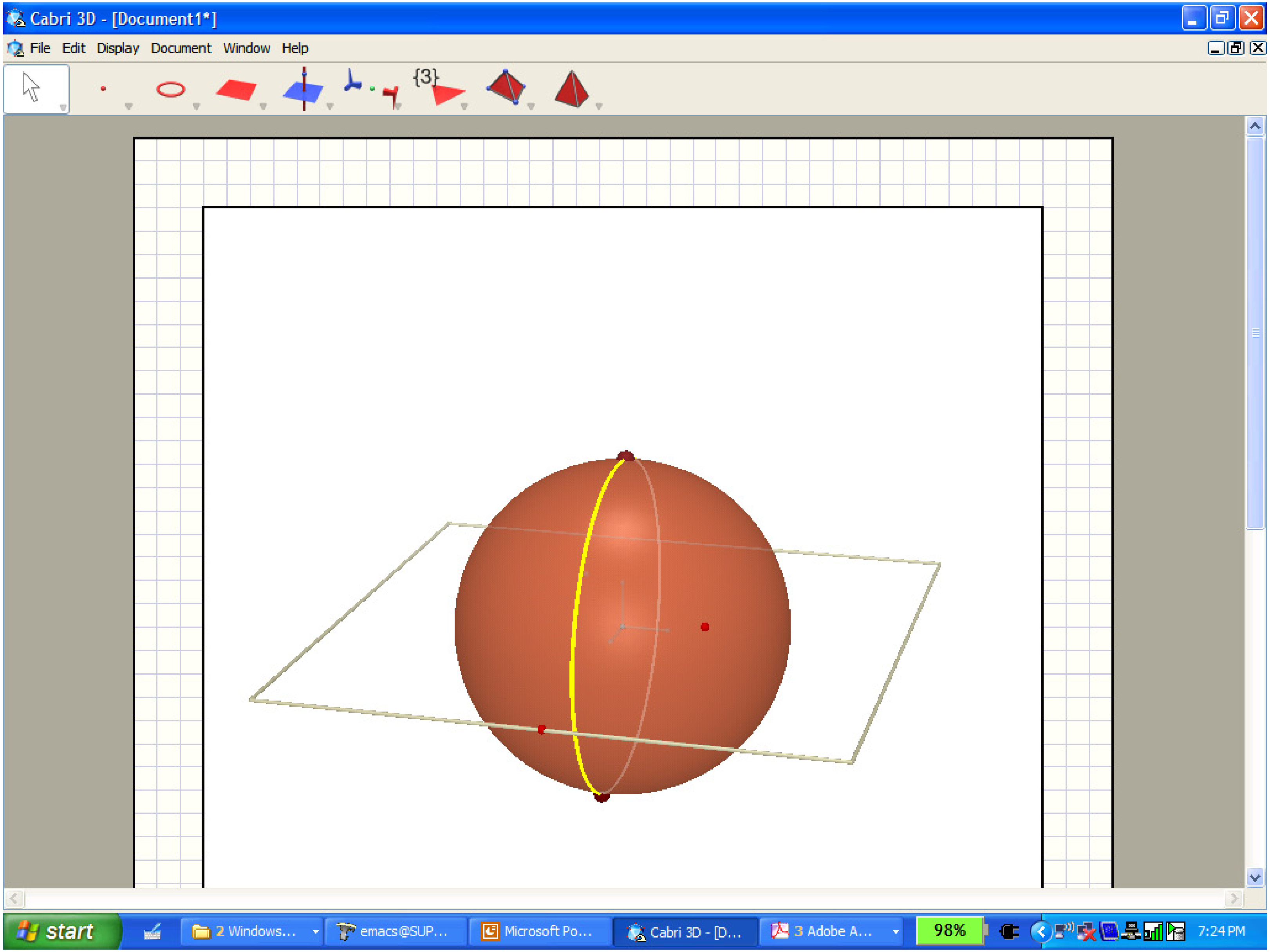
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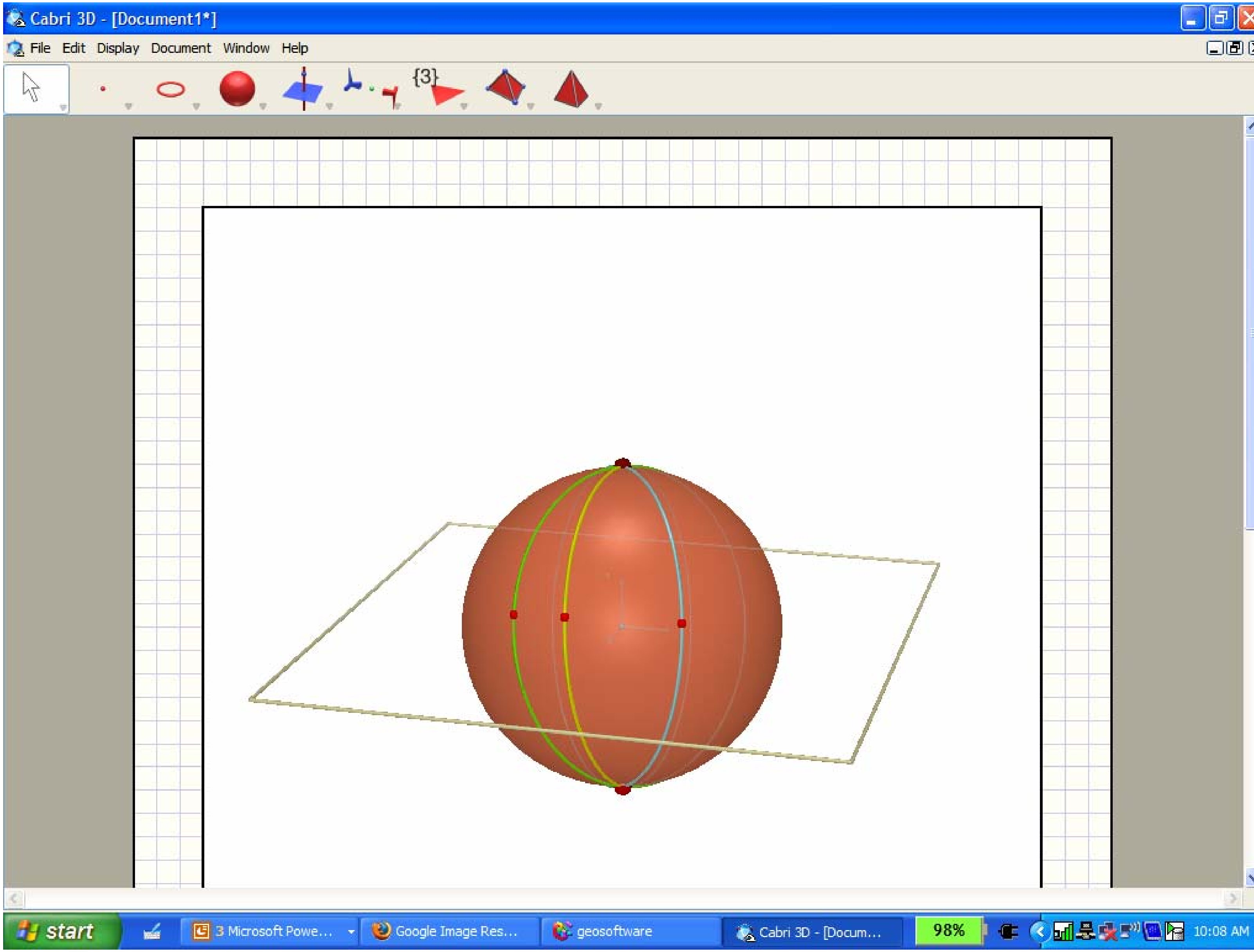
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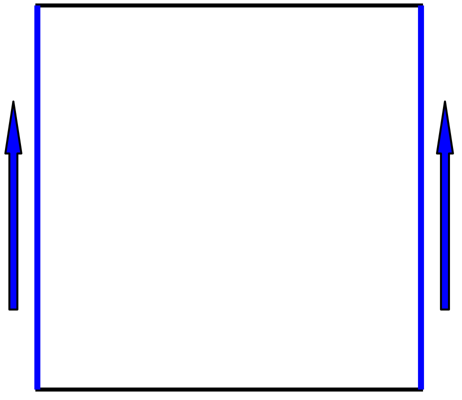
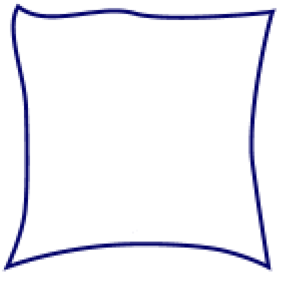
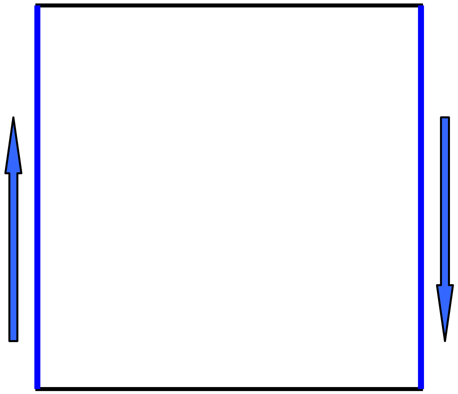
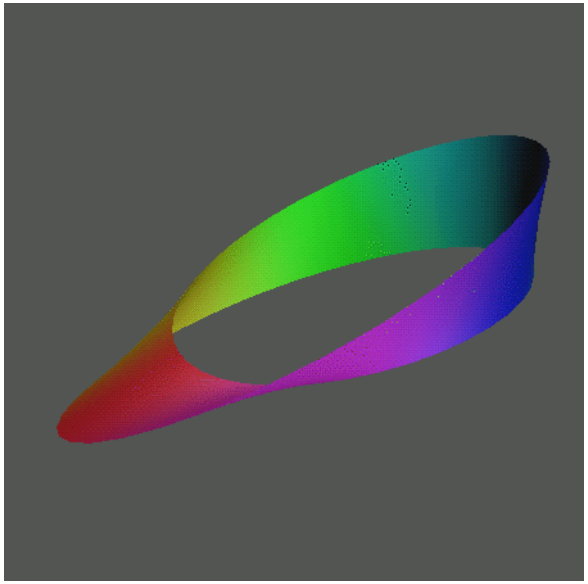
# “Gluing” spaces



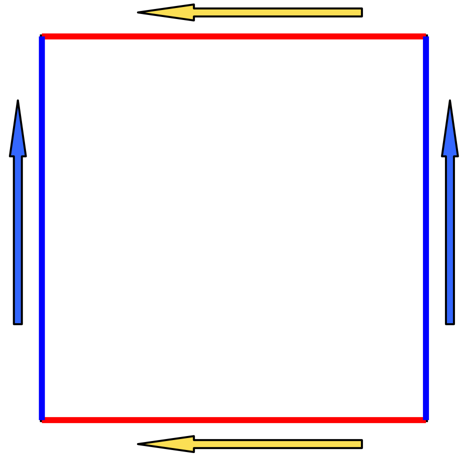


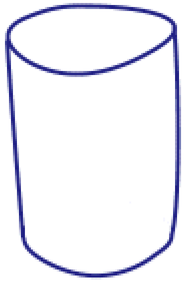
# Möbius band





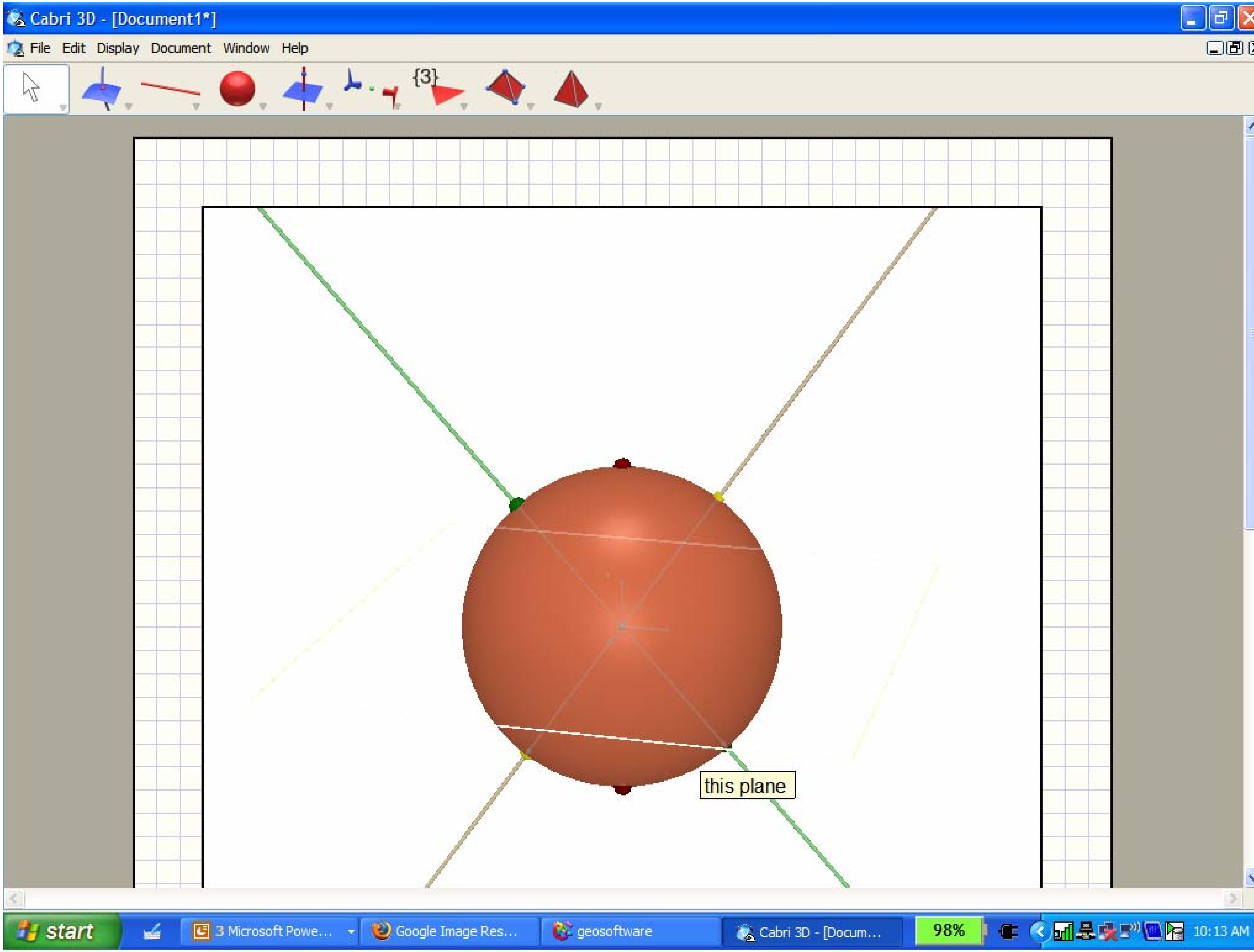
# Torus





# “Fixing” a sphere: Real projective plane

- Points are pairs {(x,y,z), (-x,-y,-z)}
  - You are gluing antipodal points
- Lines are sets of points {(x,y,z), (-x,-y,-z)} that are parts of great circles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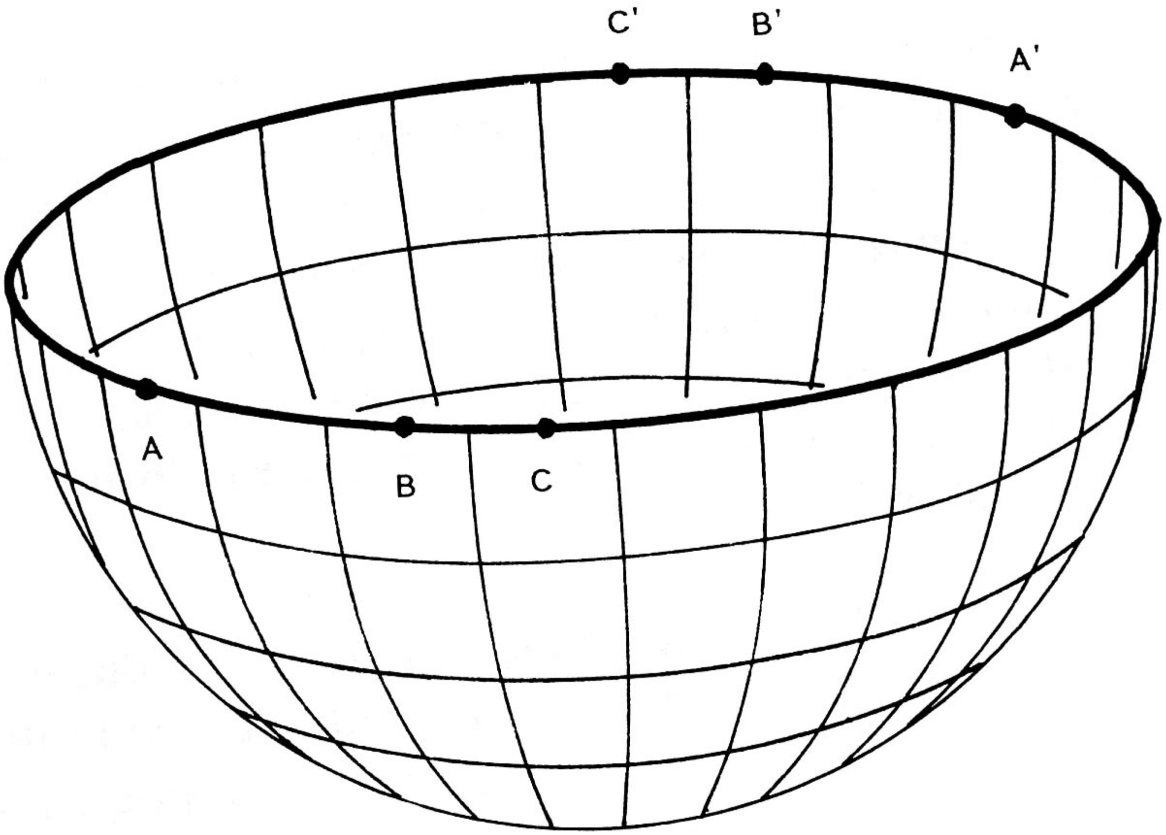
# Projective plane



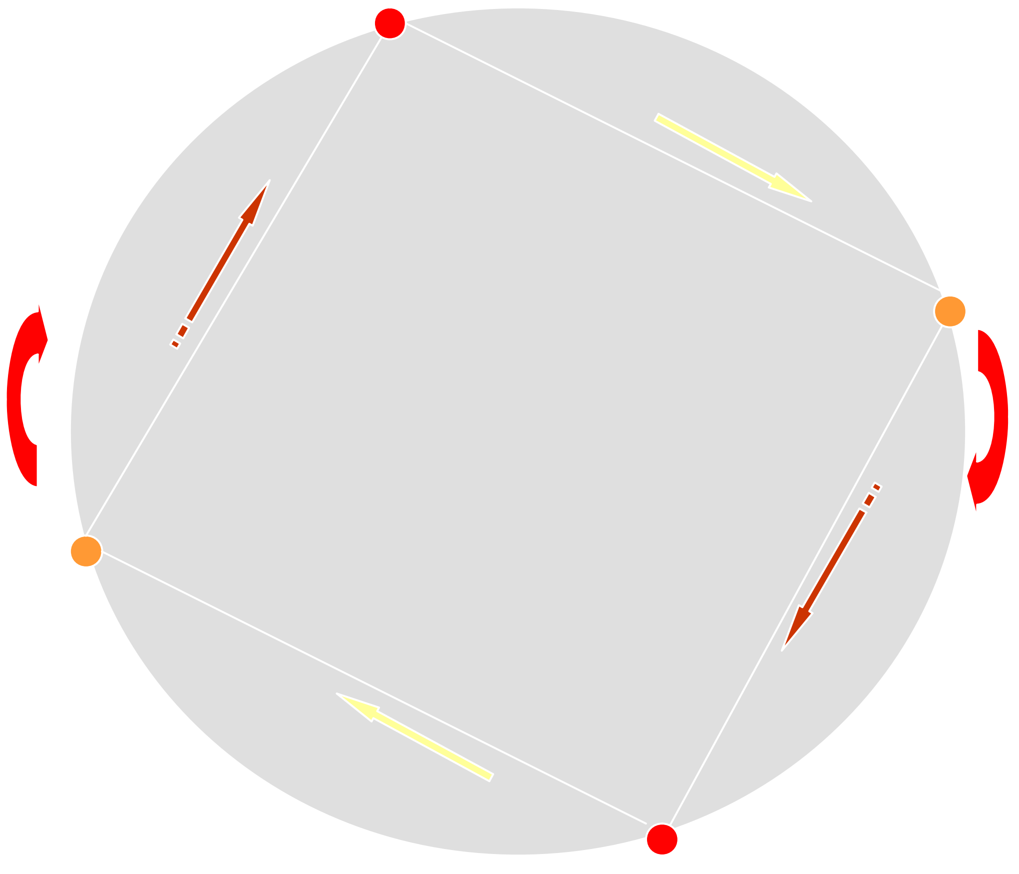

Another way define $P^2$ is to say it is a hemisphere where the antipodal points on the rim are identified.

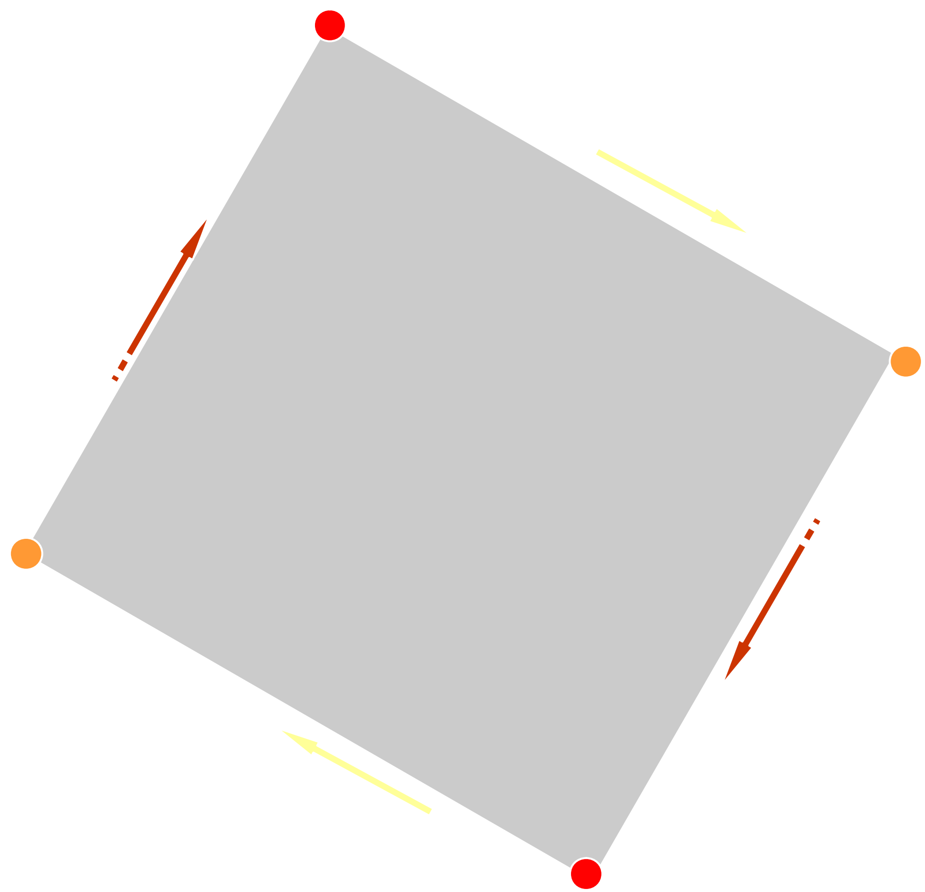
# Model #4: $P^2$

- The real projective plane is a model for incidence geometry

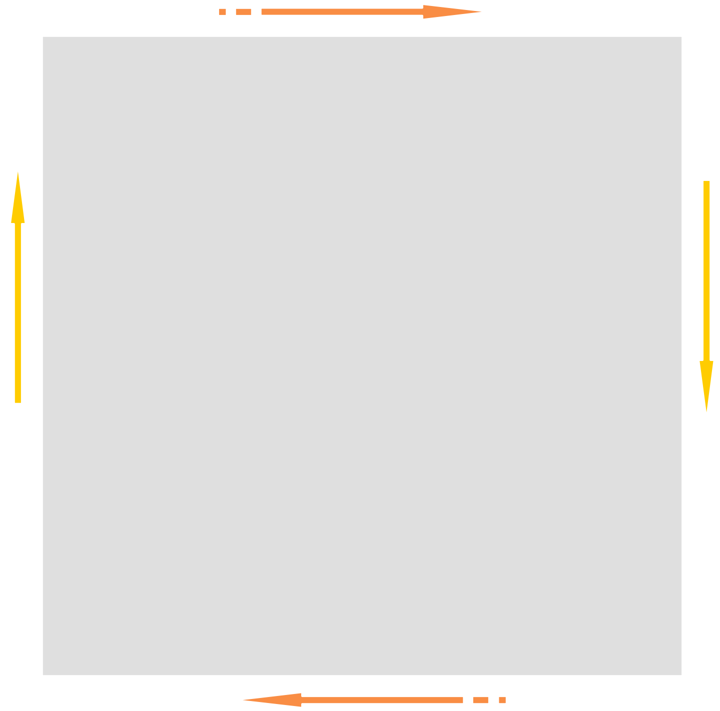
- It satisfies elliptic parallel property:
  - For every line *l* and every point P not on *l* there is no line passing through P which is parallel to *l*.

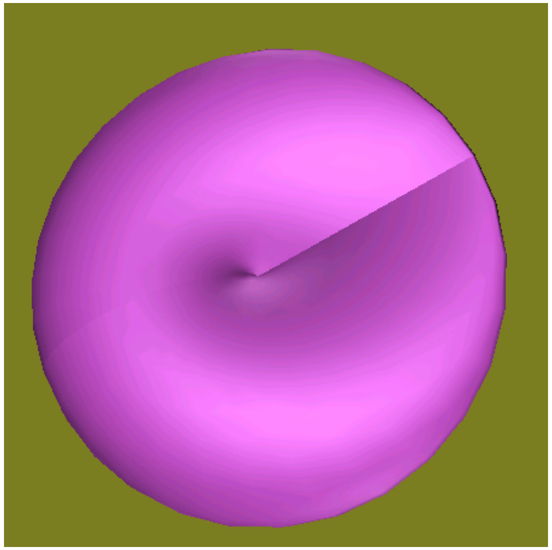
Some topological considerations

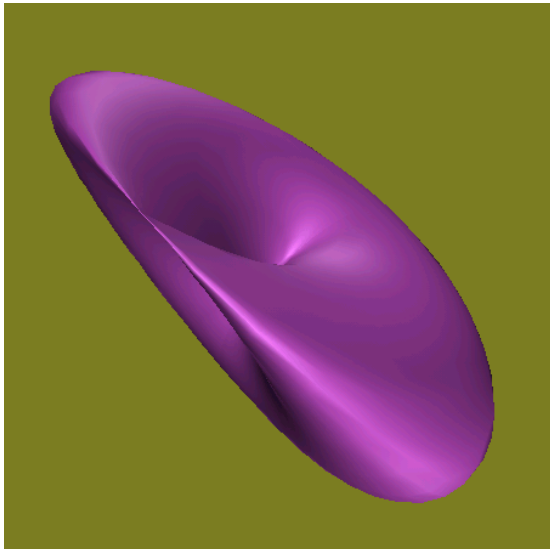


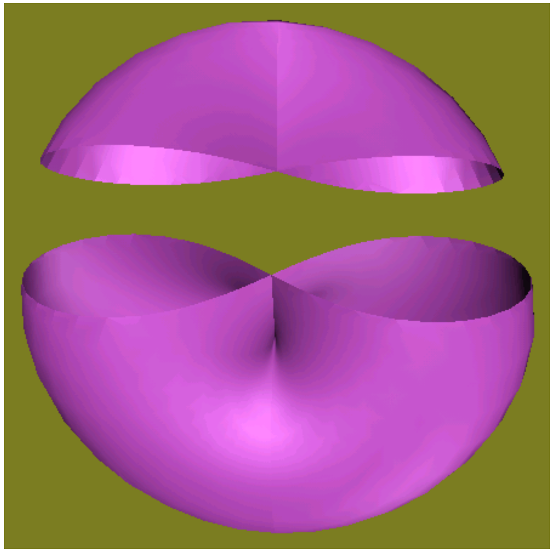


# Another way to think of $P^2$









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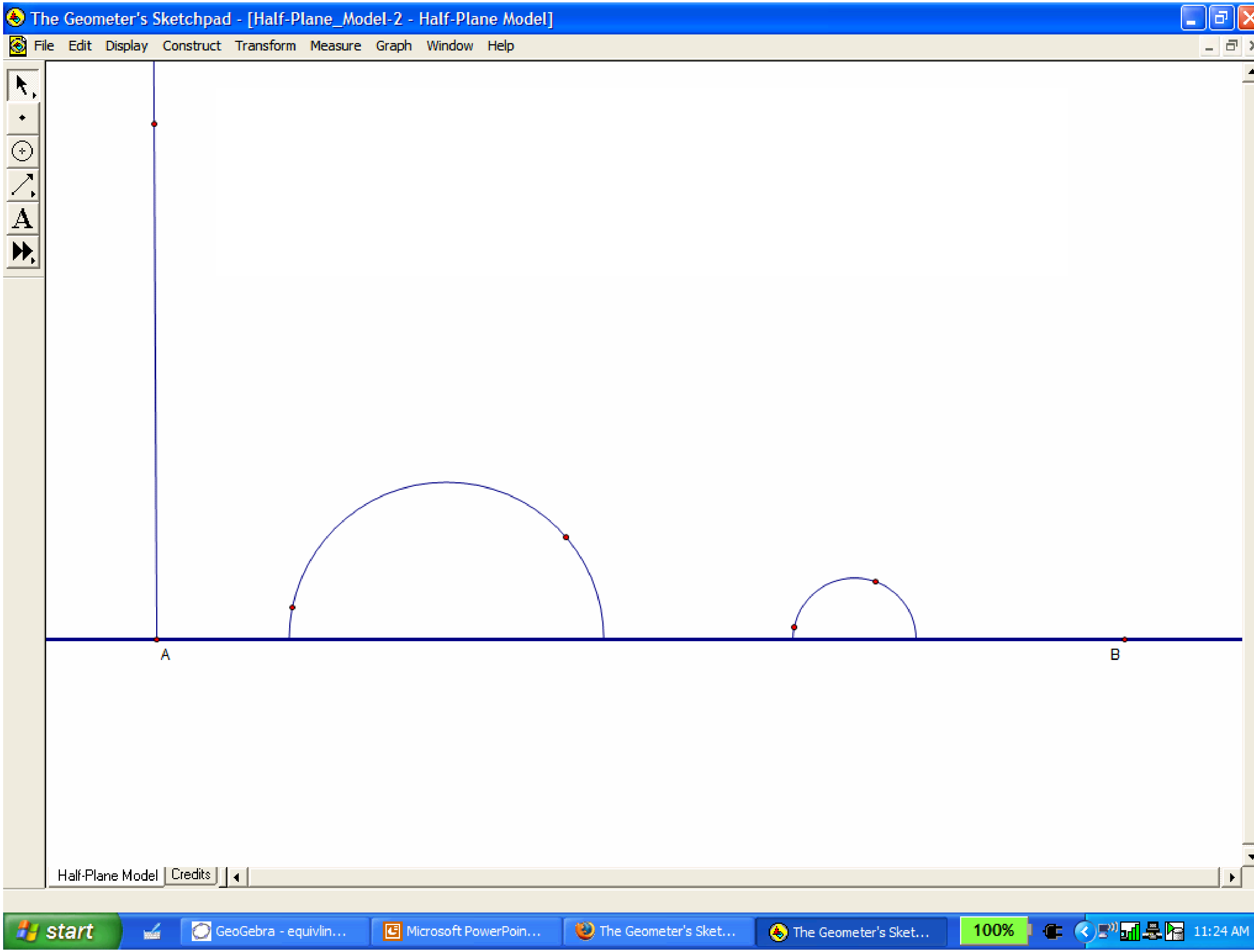
# Hyperbolic plane (the upper half plane model)

- Points are ordered pairs of real numbers $(x, y)$, where $y > 0$.

- Lines are
  - Subsets of vertical lines that consist of points $(x, y)$, with $y > 0$
  - Semicircles whose centers are points $(x, 0)$, where $x$ is a real number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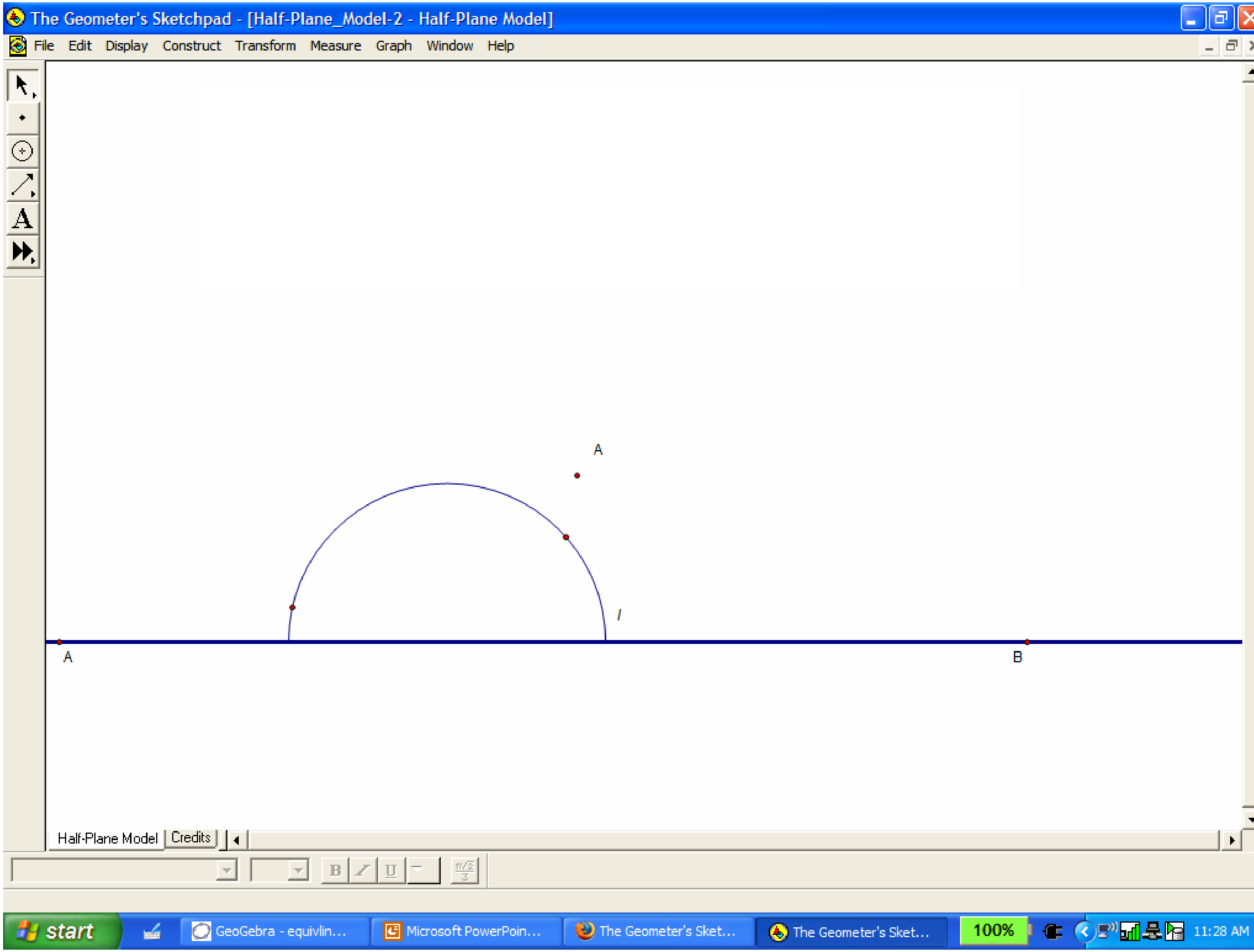
# Model #5: $H^2$

- Hyperbolic plane is also a model of incidence geometry

- It satisfies hyperbolic parallel postulate:
  - For every line *l* and every point P not lying on *l* there are at least two lines that pass through P and are parallel to *l*.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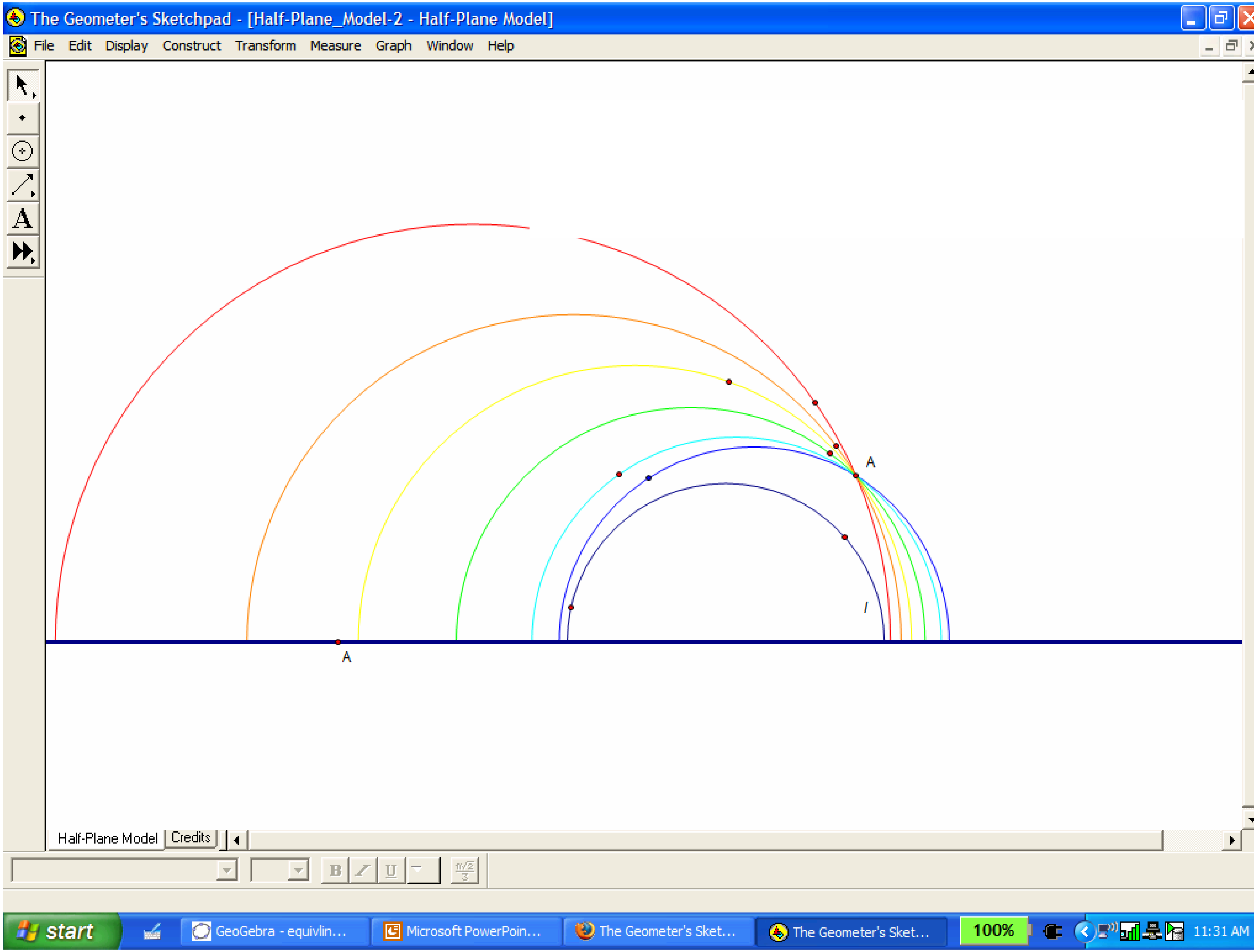
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