Axioms of incidence

And some theorems

Axioms of incidence

- ***I1*** – For every point P and for every point Q not equal to P there exists a unique line *l* incident with P and Q.

- Discuss how each of the following differs from ***I1:***
  1. There exists a line through at least two points.
  2. If you have two points, you can draw one line through them.
  3. There exist two distinct points P, Q such that they both lie on a unique line l.
  4. There exists a point P and there exists a point Q and there exists a line l such that P lies on l and Q lies on l.
  5. For all lines l and for all points P and Q on l, there is no line m such that P and Q lie on m.

- ***I1*** – For every point P and for every point Q not equal to P there exists a unique line *l* incident with P and Q.
- ***I2*** – For every line *l* there exist at least two distinct points incident with *l*.
- ***I3*** – There exist three distinct points with the property that no line is incident with all three of them.

Proposition 2.0. In incidence geometry, there is a line.

- Proof: By Axiom I-3, there exist three distinct points P, Q and R. Since $P \neq Q$, by Axiom I-1, there exists a line through P and Q. This is the desired conclusion.

Remark: This is an existence theorem. You are trying to show that some object exist. Possible approach: produce a candidate and show that it does or is what you want it to do or be. *Direct proof*

Proposition 2.1: If *l* and *m* are distinct lines that are not parallel, then *l* and *m* have a unique point in common.

- Proof: Let *l* and *m* be distinct lines that are not parallel. By definition of parallel lines, there exists a point P that lies on both *l* and *m*. (**We've simply stated the negation of the definition of parallel lines**) We wish to prove that P is the only such point; i.e., there is no other point Q lying on *l* and *m*. We will prove this by contradiction. (**We will show this by contradiction: this means assume the opposite is true, and derive a statement that contradicts a known fact or a previous step in the proof). Suppose contrary that there is a point Q such that $Q \neq P$ and Q lies on both *l* and *m*. By axiom ***I-1***, there is only line passing through P and Q. Since *l* passes through P and Q and *m* passes through P and Q, we must have $l = m$. This contradicts our assumption that $l \neq m$. (**Our assumption led to this contradiction; therefore the assumption is false**). Therefore there is no other point Q lying on *l* and *m*. We conclude that *l* and *m* have a unique point in common, namely P.