
Negation

■ Q: What is the negation of:

□ For all x , $P(x)$

There exists x such that not $P(x)$

□ There exists y such that $Q(y)$

For all y not $Q(y)$

Exercise

- Negate the following statement

S:=Every equilateral triangle is isosceles.

If you are not sure, then write the statement in the form:

For every (there exists) x , $P(x)$

and negate that statement.

S:= For every triangle x, (if x is equilateral, then x is isosceles).

$$\sim(\forall \mathbf{x}, \mathbf{P}(\mathbf{x})) \text{ means } (\exists \mathbf{x}, \sim \mathbf{P}(\mathbf{x}))$$

\sim S:= There exists a triangle x such that not (if x is equilateral, then x is isosceles).

Remark: $\sim(\mathbf{P} \Rightarrow \mathbf{Q})$ is logically equivalent to $\mathbf{P} \wedge \sim \mathbf{Q}$

\sim S:= There exists a triangle x such that x is equilateral and x is not isosceles.

Exercise

- Negate Euclid's parallel postulate:

For every line l and for every point P not lying on l there exists a unique line m such that P lies on m and m is parallel to l .

Hint: Unique means “at least one and no more than one”.

- There is a line l and a point P not on l such that either there is no line parallel to l passing through P or there is more than one line through P parallel to l .
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If (hypothesis) then (conclusion)

- We will be interested in this statement when the truth value of the hypothesis H is T(ue).
 - What does the truth value of the conclusion C have to be so that “if H then C ” is true?
 - C has to be T!
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What is proof?

- A *proof* is a list of statements together with a justification for each of them ending in a desired conclusion.
 - Justifications:
 - By hypothesis
 - By axiom
 - By theorem (already proved)
 - By step (previously done)
 - By rule of logic
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To think about and discuss!

- Do we even know that points and lines exist?
 - What are points to lines and what are lines to points?
 - Write down a list of properties that you think lines and points should satisfy. Try to agree on at most 4 that seem to be most obvious and that best describe the geometry as you know it. Use the precise language! Be mindful as this will be YOUR set of axioms for next homework.
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Axioms of incidence

- I1 – For every point P and for every point Q not equal to P there exists a unique line l incident with P and Q .
 - I2 – For every line l there exist at least two distinct points incident with l .
 - I3 – There exist three distinct points with the property that no line is incident with all three of them.
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- Proposition 2.0. In incidence geometry, there is a line.

Remark: This is an existence theorem. You are trying to show that some object exist. Possible approach: produce a candidate and show that it does or is what you want it to do or be. *Direct proof*
