Class #35

More angle sum

GOAL:

- Theorem ♦: In hyperbolic geometry every triangle has angle sum less than 180º.

- Theorem ♦: In Euclidean geometry every triangle has angle sum of 180º.

If you had these, could you prove Theorem ♦ and Theorem ♦?

- Theorem 5.2: If there is a triangle whose angle sum is not 180º then no triangle has angle sum 180º.
- Theorem 5.3: No triangle in neutral geometry can have angle sum greater than 180º.

- Theorem 5.4: If there is a triangle with angle sum 180º, then all triangles have angle sum 180º.
- Theorem 5.5: A rectangle exists iff EPP holds.
- Theorem 5.6: If a rectangle does not exist, there is a triangle with angle sum less than 180º.
- Theorem 5.7: If a rectangle exists, then there are arbitrarily large rectangles. (*proved*)
- Theorem 5.8: If a rectangle exists, then for any right triangle ΔXYZ (with right angle at X), there is a rectangle □DEFG such that DE>XY and DG>XZ. (*proved*)
- Theorem 5.9: If a rectangle exists, then every right triangle has angle sum of 180º.

- Theorem 5.10: If every right triangle has angle sum 180º, then every triangle has angle sum 180º.
- Theorem 5.11: If there is a right triangle with angle sum 180º, then a rectangle exists. (*proved*)

Theorem ♦: In hyperbolic geometry every triangle has angle sum less than 180º.

Every triangle with angle sum less than 180º.

⇑ T5.2 and T5.3

There is a triangle with angle sum less than 180º.

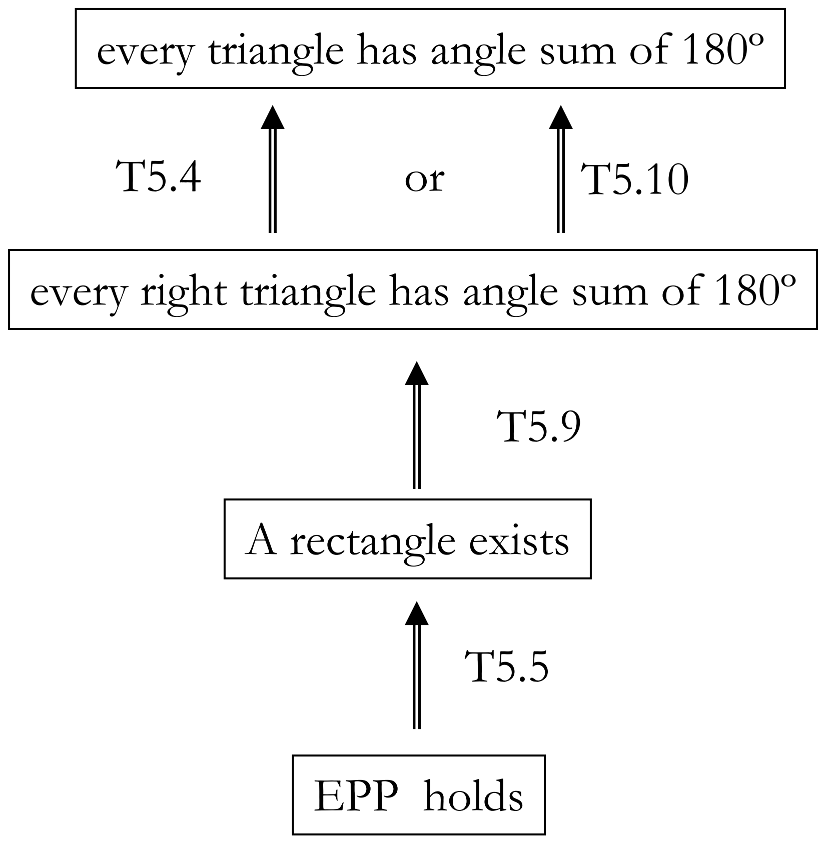
⇑ T5.6

A rectangle does not exist

⇑ Contrapositive of T5.5

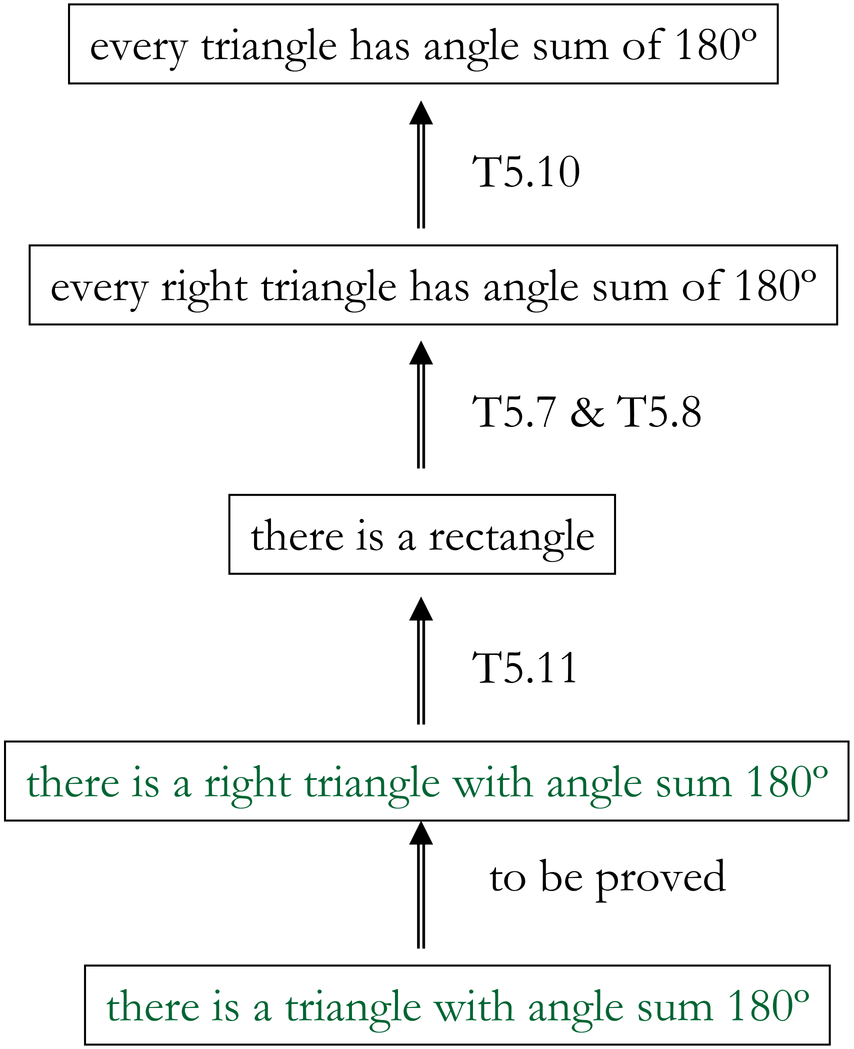
In hyperbolic geometry HPP holds $\Rightarrow$ EPP does not hold

Theorem ♦: In Euclidean geometry every triangle has angle sum of 180°.

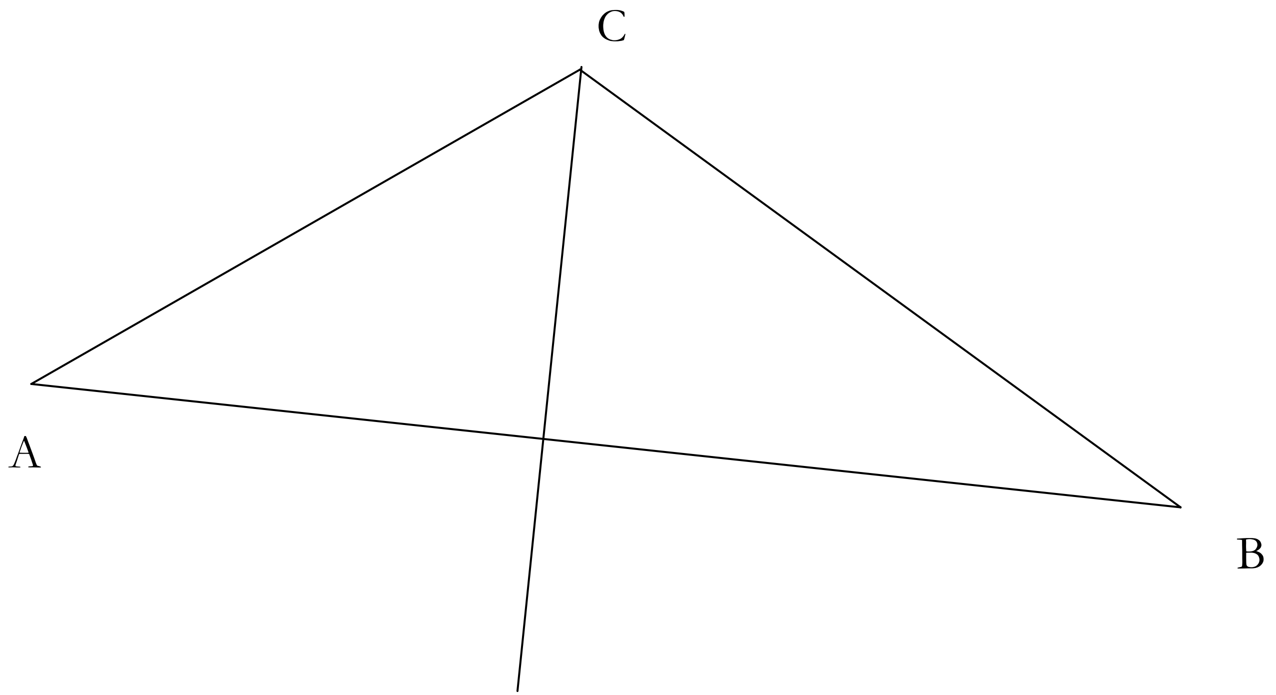


Theorem 5.4: If there is a triangle with angle sum 180º, then all triangles have angle sum 180º.

- Proof:
  - If there is a right triangle whose angle sum is 180º, then a rectangle exist (Theorem 5.11)
  - This implies that there are arbitrarily large rectangles, and consequently every right triangle has angle sum 180º. (Theorem 5.8, 5.9)
  - Therefore all triangles have angle sum 180º (Theorem 5.10).
- Remains to be shown: If there is a triangle with angle sum 180º, then there is a right triangle whose angle sum is 180º. How could you do that?



Prove: If there is a triangle with angle sum 180°, then there is a right triangle whose angle sum is 180°

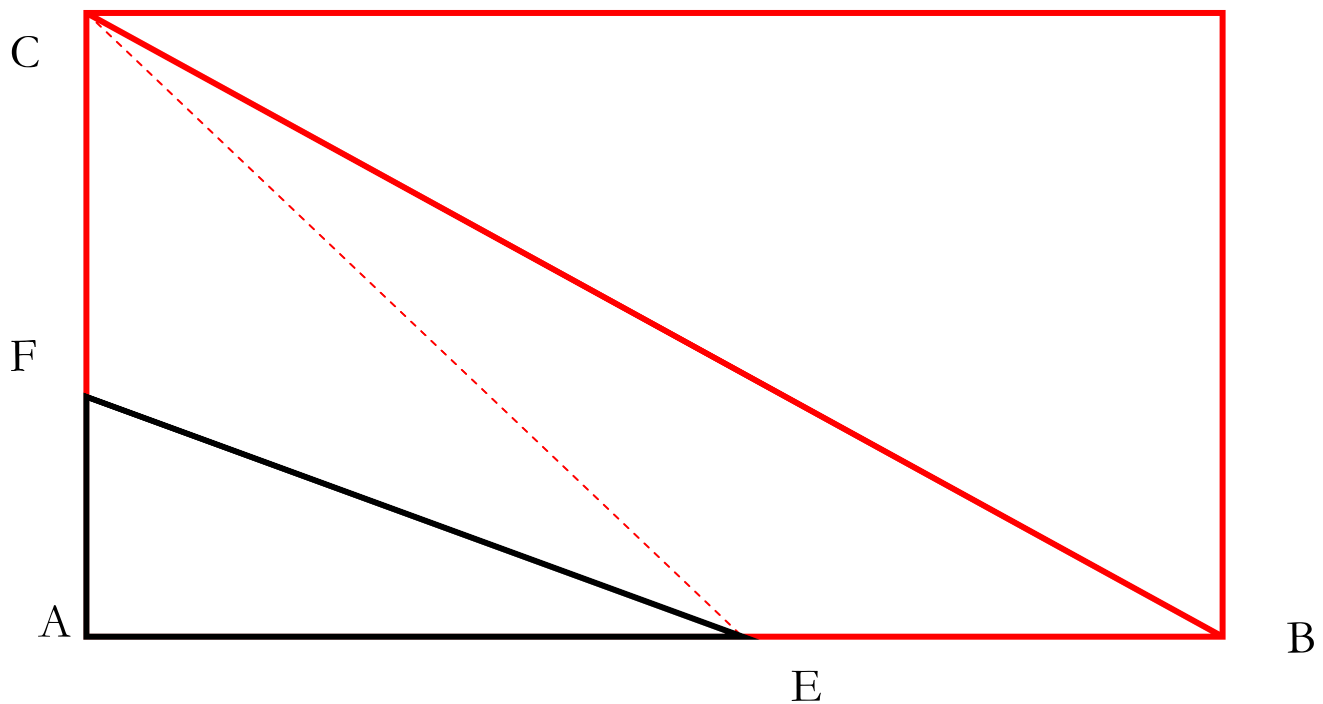


$\alpha+\beta+\gamma= 180°$

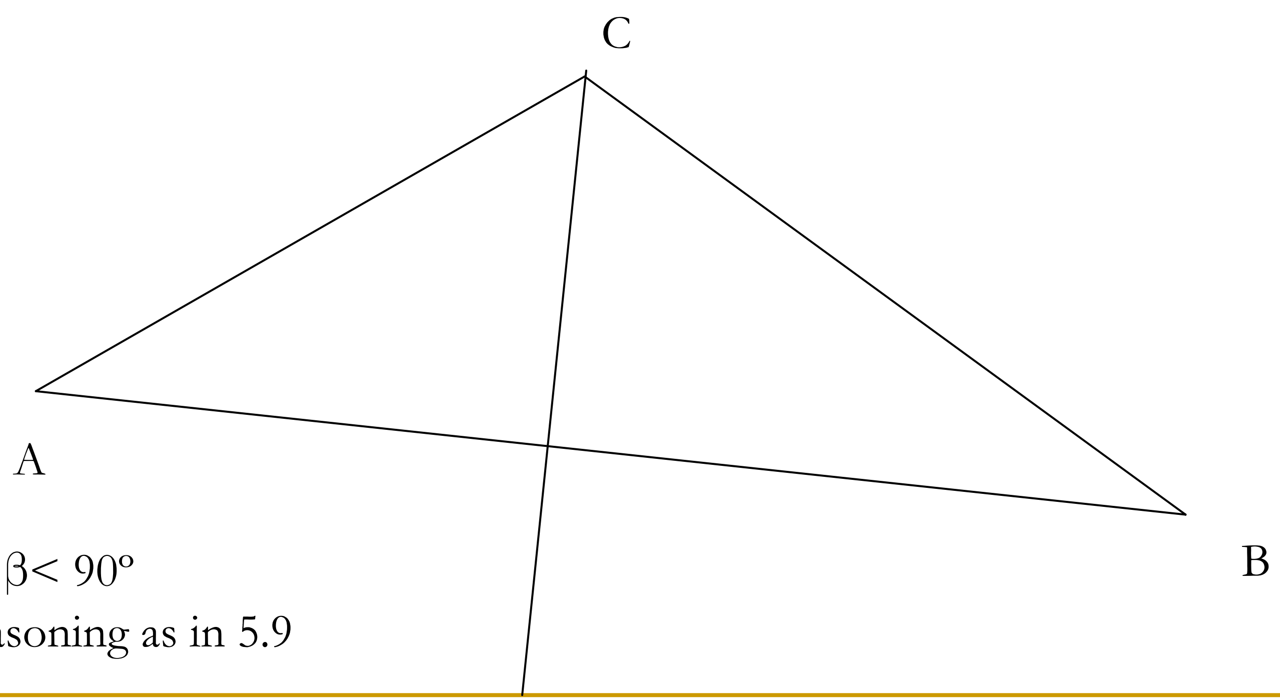
WLOG $\alpha<90°$, $\beta< 90°$

Theorem 5.9 : If a rectangle exists, then every right triangle has angle sum of 180º.

Strategy: Use 5.8 to put the triangle into a large rectangle.
Consider the ΔABC and show that its angle sum is 180º. Then show that
This means that each triangle ΔAEC and ΔEBC has to have angle sum 180º. Apply
The same reasoning to ΔAEC to show that ΔAEF must have angle sum 180º as well.



Theorem 5.10: If every right triangle has angle sum 180°, then all triangles have angle sum 180°.



WLOG $\alpha < 90°$, $\beta < 90°$

Apply similar reasoning as in 5.9