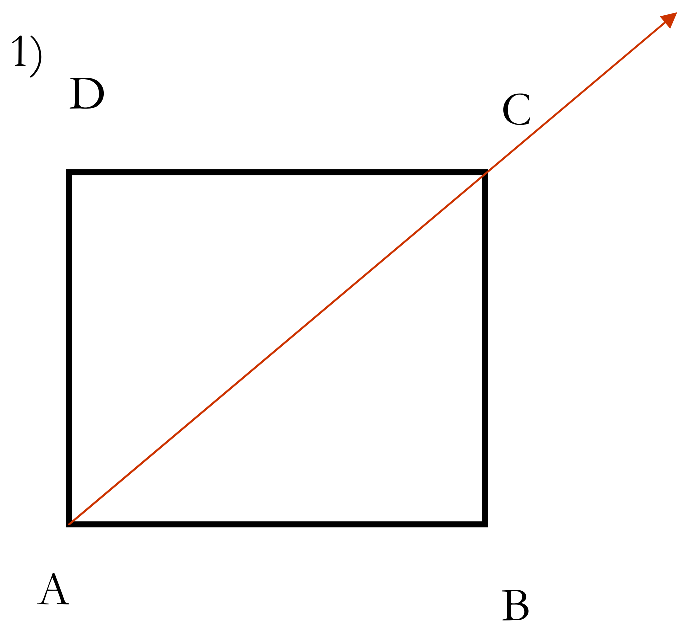
Class #33

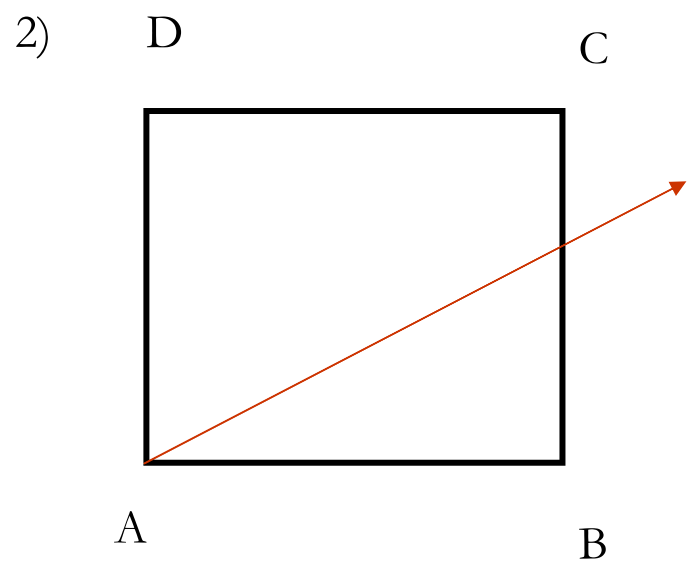
Most popular response to

- What did the students want to prove?
  - The angle bisectors of a square meet at a point.
- A *square* is a convex quadrilateral in which all sides are congruent and all angles are right angles.

Claim: The angle bisectors of a square meet at a point.

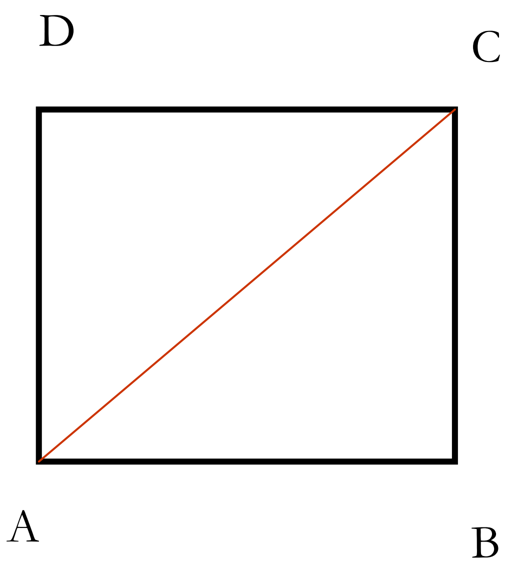
- Proof: Consider the angle bisector of $\sphericalangle BAC$.





This is too complicated. Let us try a different approach.

Claim: The angle bisectors of a square meet at a point.



- Proof: Let a be the angle bisector of $\sphericalangle A$, b the angle bisector of $\sphericalangle B$, etc. Consider the triangles ΔACB and ΔACD.
- Since $AB \cong AD$, $BC \cong CD$, by SSS, we have that triangles ΔACB and ΔACD are congruent.
- By definition of congruent triangles $\sphericalangle BAC \cong \sphericalangle DAC$. This and the fact that C is in the interior of the angle $\sphericalangle BAD$ (consequence of the definition of convex quadrilateral), imply that the ray $\overrightarrow{AC}$ is the angle bisector of $\sphericalangle BAD$, that is $a = \overrightarrow{AC}$.
- Similarly, we conclude that $c = \overrightarrow{CA}$.
- By Prop 3.1. $a \cap c = AC$.

- Consideration of triangles ΔBDA and ΔBDC, would in the identical manner give us that $b \cap d$ = BD.
- We now have (using set theory)

$$a \cap c \cap b \cap d = \mathrm{AC} \cap \mathrm{BD}$$

- The intersection of the angle bisectors is the intersection of the diagonals, and we have proved that the diagonals of a convex quadrilateral intersect in a point. Therefore, the angle bisectors of a square intersect in a point.

$\blacksquare$

Most popular response to

- What did the students prove?
  - The angle bisectors of a square are the diagonals.
- Could we prove this?
- How would you rephrase it so that it is a meaningful statement?

12

1

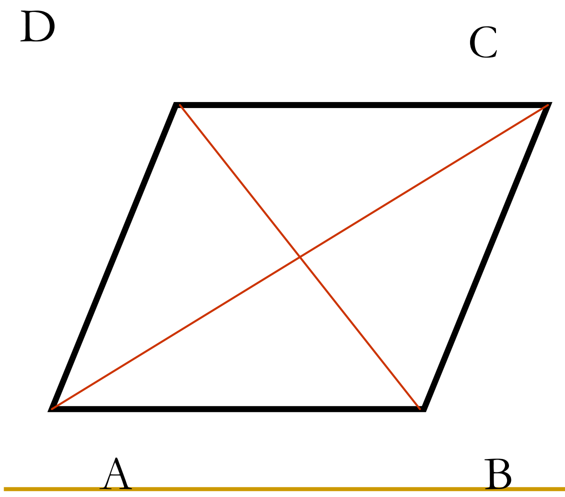
Conjectures @12

- Group 1 :
  - Def: A parallelogram is a convex quadrilateral whose opposite sides are parallel.
  - If the diagonals of a parallelogram □ABCD lie on the angle bisectors such that BD ⊂bisector(∢ABC), BD ⊂bisector(∢ADC), AC ⊂bisector(∢DCB), AC ⊂bisector(∢DAB), then all four sides DC, AB, BC and DA are congruent.
- Group 2:
  - In a parallelogram the lines defined by opposite angle bisectors are either equal or parallel.
- Group 3:
  - 1: If all four sides are congruent, the angle bisectors of opposite angles are collinear, the bisectors of adjacent angles intersect at a point and are perpendicular.
  - 2: If opposite sides are parallel, then the angle bisectors of adjacent angles are perpendicular.
  - 3: If all 4 sides are different lengths, you are screwed.
- Group 4:
  - Bisectors of adjacent angles always meet. Therefore, one angle bisector will intersect at least 2 other angle bisectors and sometimes all 3.
- Group 5:
  - If all sides of a quadrilateral are congruent, then the intersection of all 4 angle bisectors is one point.

Group 1

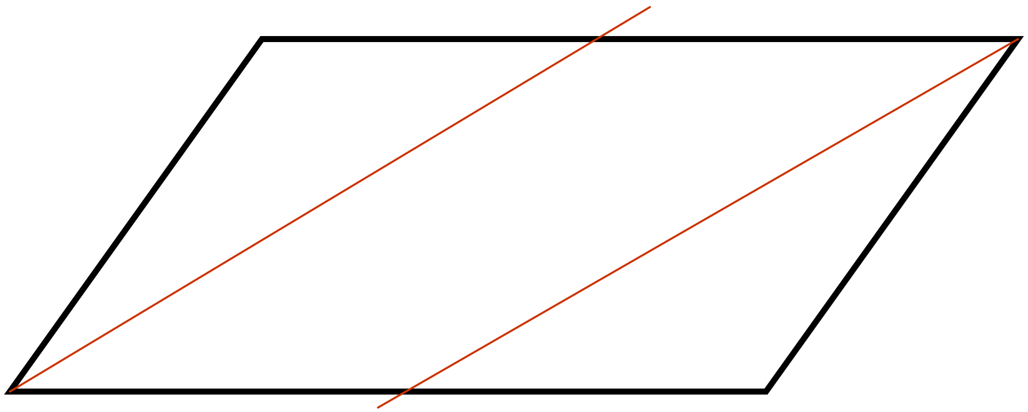
- Def: A parallelogram is a convex quadrilateral whose opposite sides are parallel.

- If the diagonals of a parallelogram □ABCD lie on the angle bisectors such that BD ⊂bisector(∢ABC), BD ⊂bisector(∢ADC), AC ⊂bisector(∢DCB), AC ⊂bisector(∢DAB), then all four sides DC, AB, BC and DA are congruent.



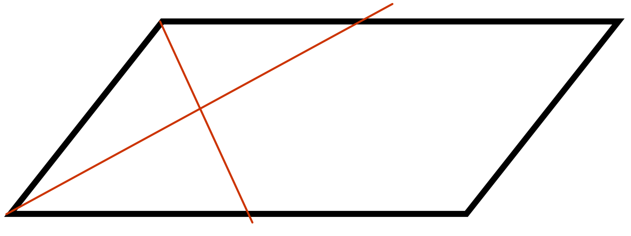
Group 2

- In a parallelogram the lines defined by opposite angle bisectors are either equal or parallel.



Group 3:

- 1: If all four sides are congruent, the angle bisectors of opposite angles are collinear, the bisectors of adjacent angles intersect at a point and are perpendicular.
  - Angle bisectors are collinear?
- 2: If opposite sides are parallel, then the angle bisectors of adjacent angles are perpendicular.



- 3: If all 4 sides are different lengths, you are screwed.

Group 4:

- Bisectors of adjacent angles always meet. Therefore, one angle bisector will intersect at least 2 other angle bisectors and sometimes all 3.

Group 5

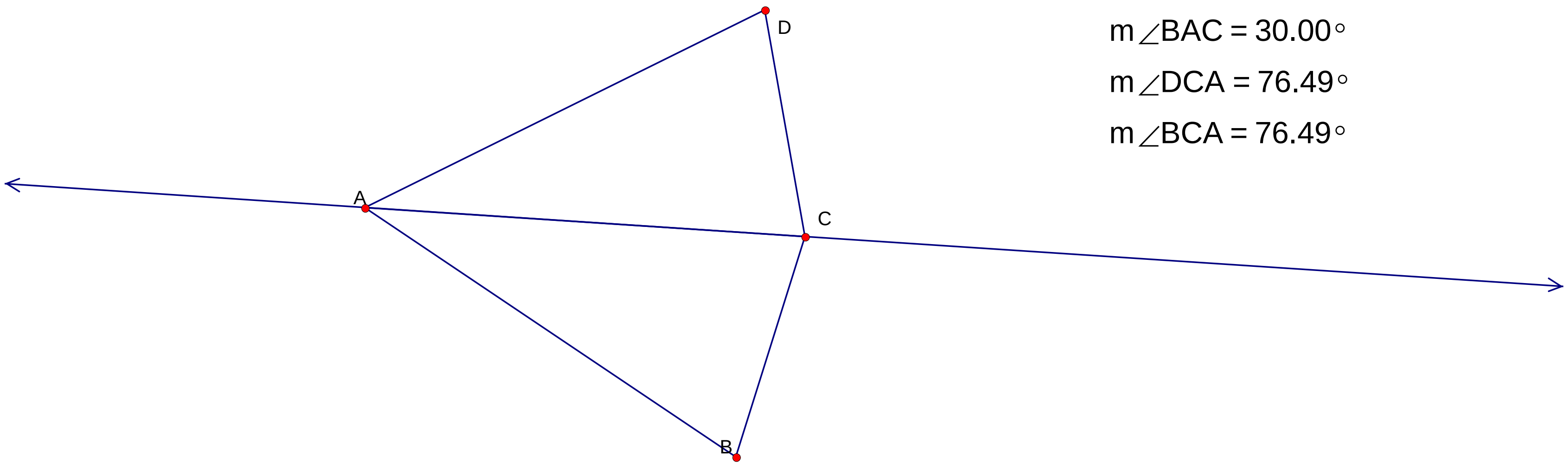
- If all sides of a quadrilateral are congruent, then the intersection of all 4 angle bisectors is one point.
- Proof:

Conjectures @1

- Group 1:
  - Given a convex quadrilateral □ABCD, if the intersection of the angle bisectors emanating from any two opposite vertices is a segment, then those segments are diagonals and opposite angles are congruent.
- Group 2:
  - If all sides of a convex quadrilateral are congruent the angle bisectors meet at a unique point in the interior of the quadrilateral.
- Group 3:
  - The angle bisector of a convex quadrilateral intersects one side of a quadrilateral not containing the vertex it originated from. If it intersects both sides, then it contains its opposite vertex.
- Group 4:
  - Given a convex quadrilateral, if the intersection of the angle bisectors of the angles formed by the opposite vertices are equal to the diagonals, then the quadrilateral is a square.
  - A square is a quadrilateral with all four sides congruent and all four angles right angles.
- Group 5:
  - If a rectangle is not a square, then the angle bisectors intersect to form a square.
  - Rectangle - quadrilateral with four right angles and opposite sides congruent.
  - Square – rectangle with all sides congruent

Group 1:

- Given a convex quadrilateral □ABCD, if the intersection of the angle bisectors emanating from any two opposite vertices is a segment, then those segments are diagonals and opposite angles are congruent.



$m\angle DAC = 30.00°$

$m\angle BAC = 30.00°$

$m\angle DCA = 76.49°$

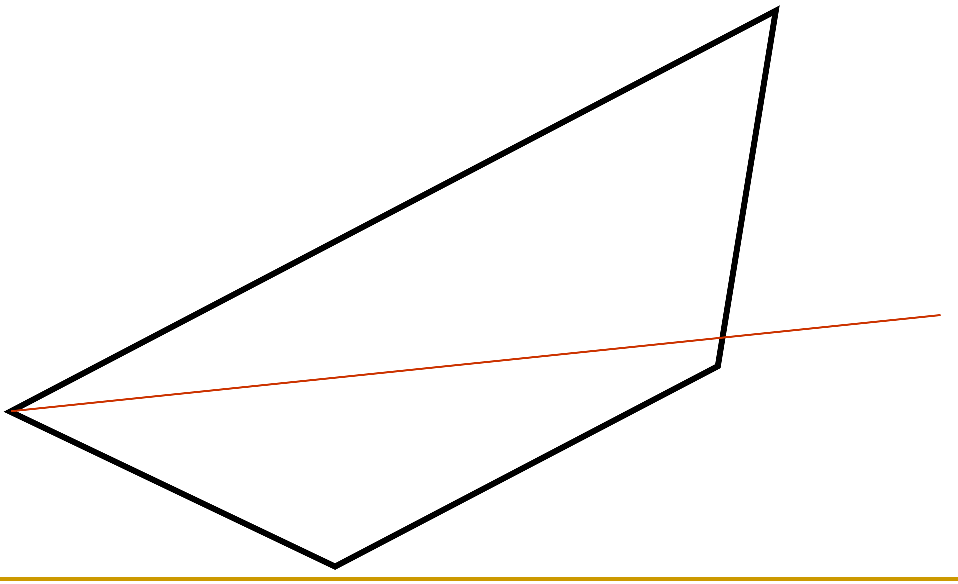
$m\angle BCA = 76.49°$

Group 2:

- If all sides of a convex quadrilateral are congruent the angle bisectors meet at a unique point in the interior of the quadrilateral.
- Proof:

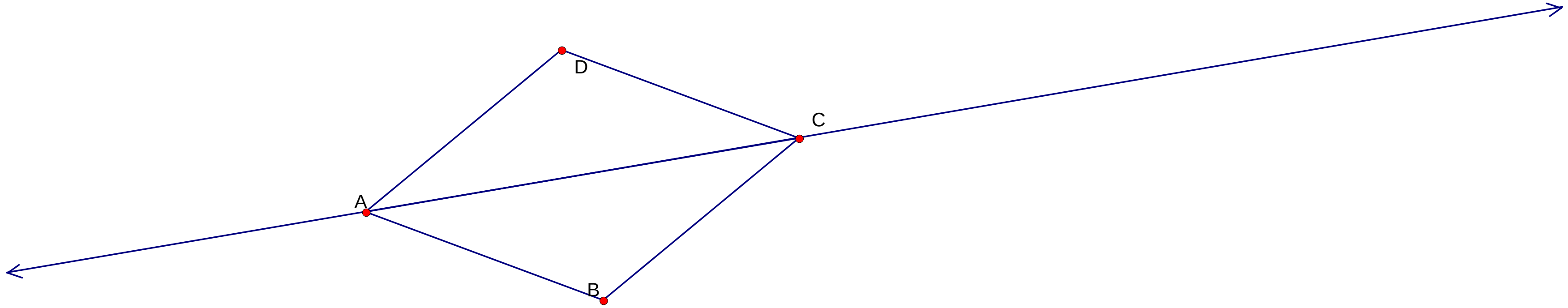
Group 3:

- The angle bisector of a convex quadrilateral intersects one side of a quadrilateral not containing the vertex it originated from. If it intersects both sides, then it contains its opposite vertex.



Group 4:

- Given a convex quadrilateral, if the intersection of the angle bisectors of the angles formed by the opposite vertices are equal to the diagonals, then the quadrilateral is a square.



$m\angle DAB = 60.00°$

AD = 4.19 cm

AB = 4.19 cm

DC = 4.19 cm

BC = 4.19 cm

Group 5:

- If a rectangle is not a square, then the angle bisectors intersect to form a square.
- Rectangle - quadrilateral with four right angles and opposite sides congruent.
- Square – rectangle with all sides congruent