Class #31

@12

G1– Little devils	G2 – False proofs	G3 – definitions	G4 – sketches	G5 – examples and counters
				Jacob
Lisa	Nese	Rachel	Kristen	Sarah
Kevin	Meg	Anthony	Matt	Mike
Jasmin	Victor	David	Jenny	Stephen
Erik	TJ	Tricia	Eddy	Sam

@1

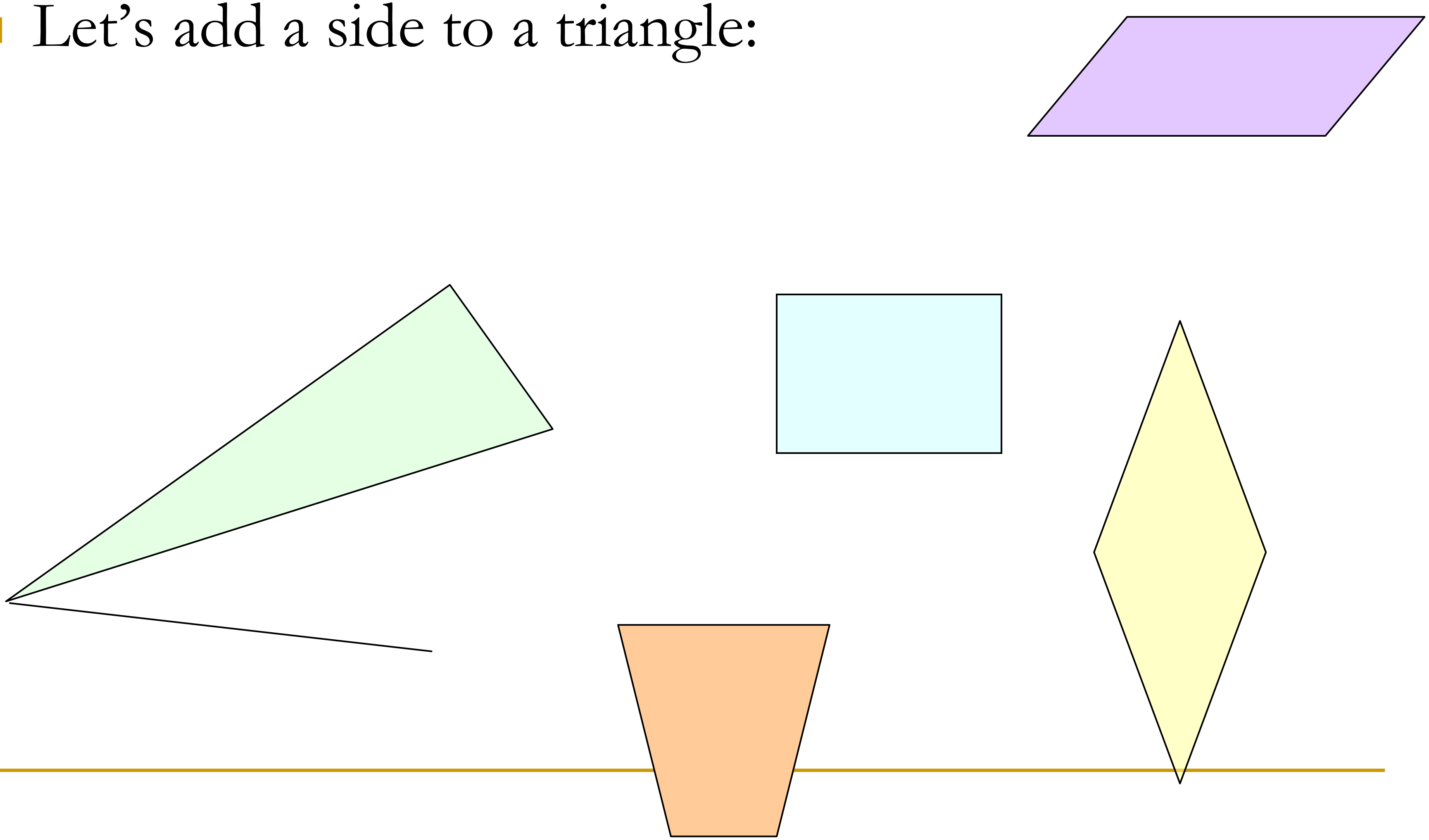
G1 – Little devils	G2 – sketches	G3 – false proofs	G4 – examples and counters	G5 – definitions
Amanda	Rachel	Sarah R	Julia	Laura
Sarah Y	Josh	Laurence	Robert	Matt
Whitney	Sarah C	Edgar	Sarah F	Ann
William	Nikki	Adam	Jim	Ping
Yolanda	Sahar	David	Alison	

One of the findings of the colorful survey

- The winners of "So saaaad":
  I. Have we covered enough for me to teach high school/topics related to high school geo – 10
  II. Something more complicated than a Δ – 8
  III. More hyperbolic geometry – 7

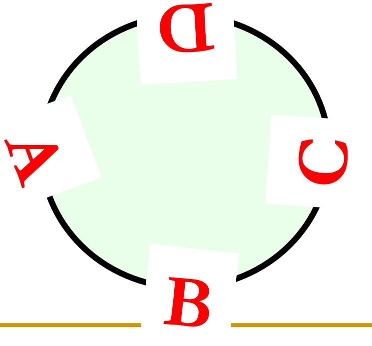
The winner and the second best:

- Let's add a side to a triangle:



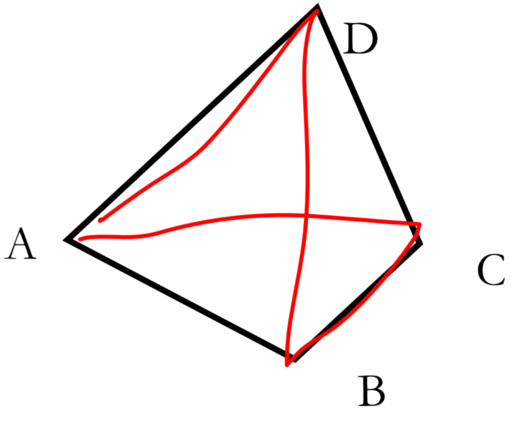
Define quadrilateral

- Remember: A *triangle* is the union of segments AB, BC and AC, where A, B and C are three distinct noncollinear points.
- If A, B, C and D are four distinct points so that no three of them are collinear and such that segments AB, BC, CD and DA have either no points in common or have only an endpoint in common, then the union of these four segments is called a *quadrilateral*.
  - Quadrilateral AB∪BC∪CD∪DA will be denoted by □ABCD, and AB, BC, CD and DA are its *sides*.
  - If the two letters are "consecutive" in this notation then they are endpoints of a side.

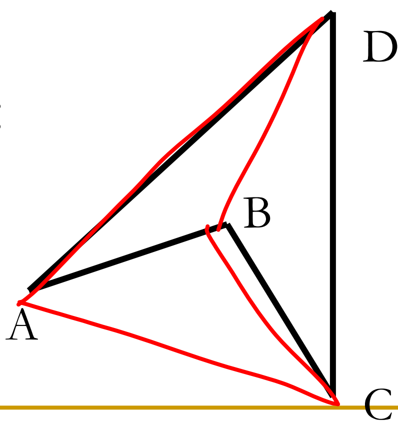


Examples and counters!

- If □ABCD is a quadrilateral, is □ACBD a quadrilateral as well?
  - Example when it is not:



  - Example when it is:

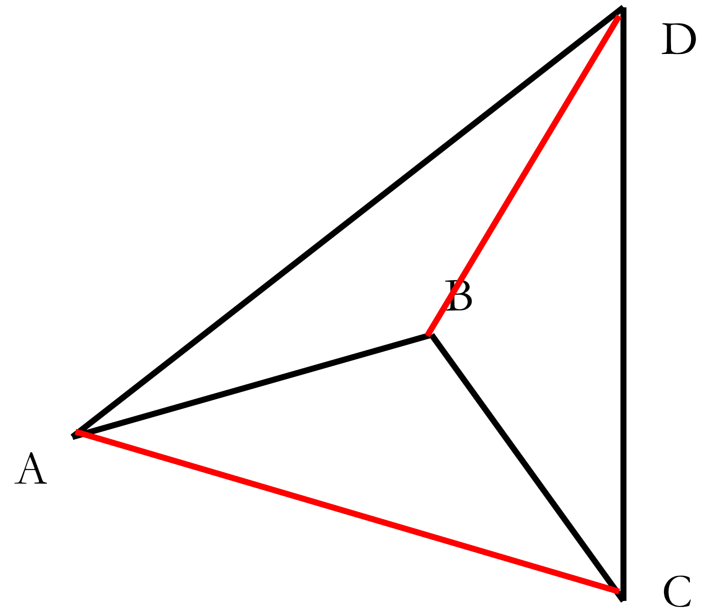


Nuts and bolts of □ABCD

- Vertices
  - Points A, B, C and D are called vertices of □ABCD.
- Adjacent vertices
  - Two vertices are adjacent if they are endpoints of a side.
- Opposite vertices
  - Two vertices are opposite if they are not adjacent.
- Adjacent sides
  - Two sides are adjacent if they have a common endpoint.
- Opposite sides
  - Two sides are opposite if they are not adjacent.
  - Segments whose endpoints are opposite vertices.
  - Alternatively, AC and BD are diagonals.

Vote: Raise your hand if you think the following statement is true:

1. The diagonals of □ABCD meet at a point.



Convex quadrilateral

- A quadrilateral is *convex* if for every pair of opposite sides the endpoints of each side are on the same side of the line determined by the endpoints of the other.

We now define:

- Angles
  - The angles ∢BAD, ∢ABC, ∢BCD, and ∢CDA are the angles of □ABCD.
- Interior of □ABCD
  - The intersection of the interiors of its angles.

We could now prove:

- Theorem 4.6: Diagonals of a convex quadrilateral meet at a point.

What can you say about angle bisectors of a convex quadrilateral.

- **Directions**: With your group come up with a conjecture. Write it on a POST-IT, together with an outline of why you believe your conjecture is true. Once done, try to prove it. Here is a sample.

e, Pat, Michael :

Conjecture : You would like to know what it is.

Intuition: Experience.

Proof: You're still reading :)

FOR FRIDAY!!!

Post your work then walk around and pick the conjecture you like the best, whether because you think is true or false is irrelevant, and write your name next to it (you can keep yours). **Make a record of the conjecture. Your task for Friday is to see whether you can prove it, or if you can find a counterexample. On Friday, as you walk in write "Proven" or "Found counterexample" next to your name.**

e, Pat, Michael :

Conjecture : You would like to know what it is.

Intuition: Experience.

Proof: You're still reading

:)

e
Matt
Sarah

Theorem: Diagonals of a convex quadrilateral meet at a point.

Sketch of proof: Point C is in the interior of angle $\sphericalangle BAC$ (CD is opposite side to AB, so C and D lie on the same side of $\overleftrightarrow{AB}$ (by definition of convex quadrilateral); BC is opposite side to AD, so B and C lie on the same side of $\overleftrightarrow{AD}$). By definition of between for rays, $\overrightarrow{AC}$ is between rays $\overrightarrow{AB}$ and $\overrightarrow{AD}$. Crossbar theorem guarantees that $\overrightarrow{AC}$ intersects segment BD, let's say at point P. We have either A*P*C or A*C*P.

- If the former is the case then AC intersects BD and we're done.
- If the latter is the case, then C is in the interior of $\sphericalangle BDA$ (Prop 3.7), so ray $\overrightarrow{DC}$ intersects AB (Crossbar), which contradicts the fact that A and B are on the same side of line CD (convex quadrilateral).