Class #30

Class@12

G1– Little devils	G2 – False proofs	G3 – definitions	G4 – sketches	G5 – examples and counters
				Jacob
Lisa	Nese	Rachel	Kristen	Sarah
Kevin	Meg	Anthony	Matt	Mike
Jasmin	Victor	David	Jenny	Stephen
Erik	TJ	Tricia	Eddy	Sam

Class@1

G1 – Little devils	G2 – sketches	G3 – false proofs	G4 – examples and counters	G5 – definitions
Amanda	Rachel	Sarah R	Julia	Laura
Sarah Y	Josh	Laurence	Robert	Matt
Whitney	Sarah C	Edgar	Sarah F	Ann
William	Nikki	Adam	Jim	Ping
Yolanda	Sahar	David	Alison	

Theorem 4.3: Every angle has a bisector.

Proof:

Let $\sphericalangle BAC$ be an angle. We need to find a ray $\overrightarrow{AD}$ between rays $\overrightarrow{AB}$ and $\overrightarrow{AC}$, such that $\sphericalangle BAD \cong \sphericalangle DAC$.

Let C' be the unique point on the ray $\overrightarrow{AC}$ such that AC' ≅ AB (by axiom C1). Then ΔBAC' is an isosceles triangle. Let D be the midpoint of BC', so that AD is a median.

Triangles ΔBAD and ΔC'AD are congruent by SSS, so the corresponding angles are congruent: $\sphericalangle BAD \cong \sphericalangle DAC$. Hence, $\overrightarrow{AD}$ is the angle bisector of $\sphericalangle BAC$.

Part of hw10: Prove 4.4 and 4.5
-- Defining groups beware – we need your input!!

- angle bisectors
- **Theorem 4.4: Angle bisectors in a triangle meet at a point.**
  - Proof: We need few new things for this proof:
  - Recall:
    - the proof that from a point not on a line there is a perpendicular to the line from that point.
    - Given a line, an angle was formed, then isosceles triangle whose base was a segment perpendicular to the given line.
    - Say you have an angle, and a point on its bisector. Form the perpendiculars from that point to the sides of an angle. What can you say about them? Phrase your conclusion as a statement (theorem, proposition).
    - In order to finish this we need to define a distance from a point to a line (or ray). Our defining groups are working on this. The definitions will be supplied on the homework sheet. If you want to help them you are welcome to.
  - **Theorem 4.5: A point P lies on the angle bisector of ∢BAC if and only if it is equidistant from the sides of ∢BAC.**

In all of the propositions below let ΔABC be a triangle and D a point on $\overleftrightarrow{AB}$

- Prop 4.2.1: If AC ≅ BC and CD is a median then $\overrightarrow{CD}$ is the angle bisector of ∢ACB.
  - Sketch: By definition of a median, D is a midpoint of AB, so AD≅BD. This, together with CD≅CD, AC ≅ BC , and SSS shows ΔACD≅ΔBCD. Angle ∢ACD corresponds to ∢BCD, so they are congruent by definition of congruent triangles. Since D is such that A*D*B it is in the interior of the angle ∢ACB (Prop 3.7), so ray $\overrightarrow{CD}$ is between rays $\overrightarrow{CB}$ and $\overrightarrow{CA}$, hence it is the angle bisector.
- Prop 4.2.2: If AC ≅ BC and CD is a median then CD is the altitude.
  - Sketch: By definition of a median, D is a midpoint of AB, so AD≅BD. This, together with CD≅CD, AC ≅ BC , and SSS shows ΔACD≅ΔBCD. Angle ∢ADC corresponds to ∢BDC, so they are congruent by definition of congruent triangles. These angles are supplementary as well, since A*D*B and DC is a side of both. By definition of a right angle, ∢ADC is a right angle, so CD is perpendicular to AB. Since D lies on AB, we have that CD is an altitude, by definition of altitude.
- Prop 4.2.3: If AC ≅ BC and $\overrightarrow{CD}$ is the angle bisector of ∢ACB, then CD is the altitude.
  - Sketch: Since $\overrightarrow{CD}$ is the angle bisector, we have ∢ACD ≅ ∢BCD. Since CD≅CD, and AC ≅ BC and using SAS we conclude that ΔACD≅ΔBCD. Angle ∢ADC corresponds to ∢BDC, so they are congruent by definition of congruent triangles. These angles are supplementary as well, since A*D*B and $\overrightarrow{DC}$ is a side of both. By definition of a right angle, ∢ADC is a right angle, so $\overleftrightarrow{CD}$ is perpendicular to $\overleftrightarrow{AB}$. Since D lies on $\overleftrightarrow{AB}$, we have that CD is an altitude, by definition of altitude.
- Prop 4.2.4: If CD is a median and $\overrightarrow{CD}$ is the angle bisector of ∢ACB, then the triangle is isosceles.
  - Sketch: By definition of a median, D is a midpoint of AB, so AD≅BD. Since $\overrightarrow{CD}$ is the angle bisector of ∢ACB, we know ∢ACD ≅ ∢BCD. Further, CD≅CD. Use ASS to conclude that ΔACD≅ΔBCD. **But ASS is NOT a theorem. WHY? Hint: To finish the proof use Theorem 4.5 – this becomes HW10.**
- Prop 4.2.5: If CD is a median and the altitude, then ΔABC is isosceles.
  - Sketch: By definition of a median, D is a midpoint of AB, so AD≅BD. Since CD is the altitude we have that ∢ADC and ∢BDC are right angles. Since CD≅CD, using ASA we conclude that ΔACD≅ΔBCD. By definition of congruent triangles we have AC ≅ BC, hence the triangle ΔABC is isosceles, by definition.