Class #27

Remider

- We proved Alternate interior angle theorem:
  - If two lines cut by a transversal have congruent alternate interior angles then those two lines are parallel.

Investigations

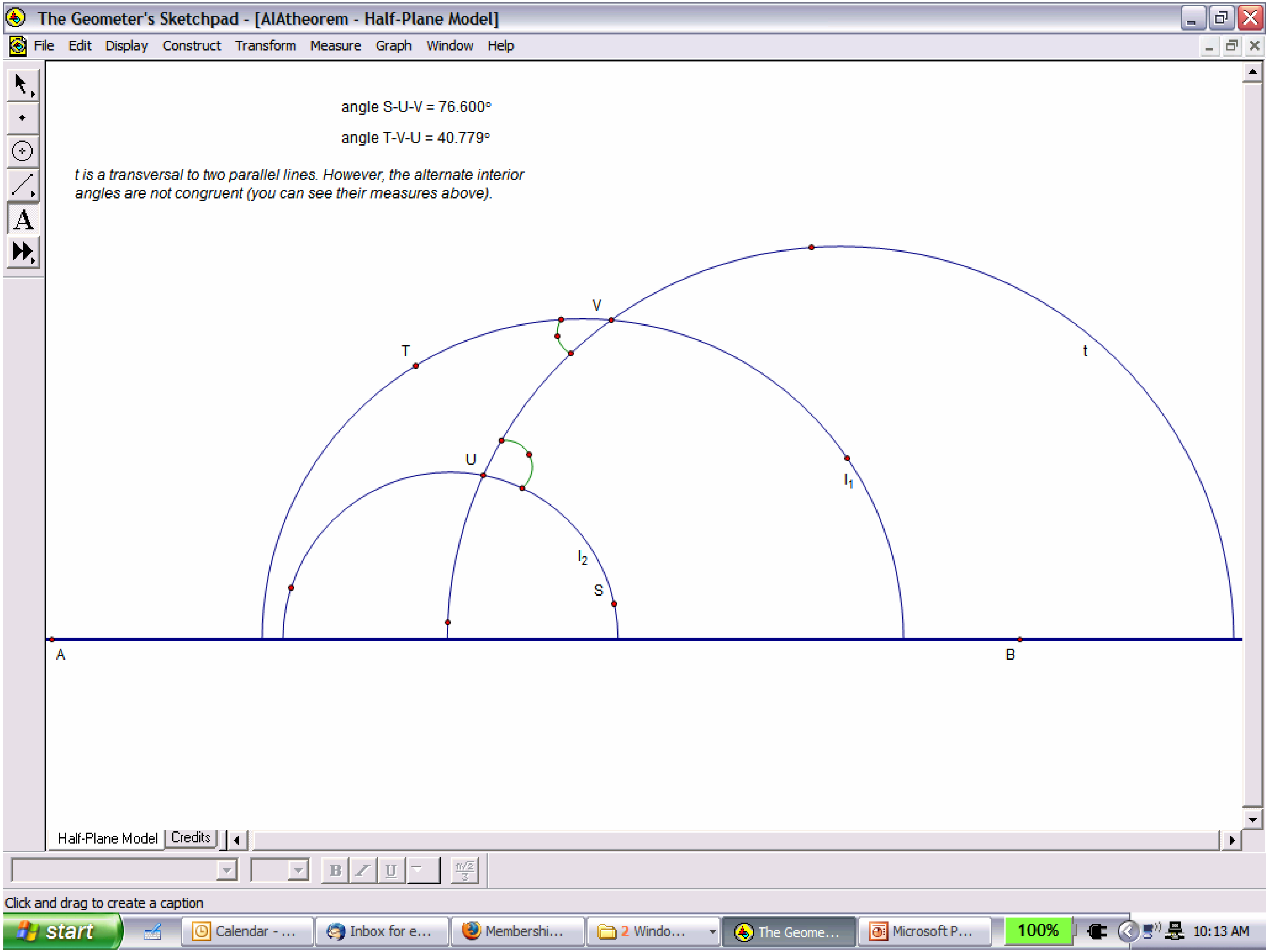
- What can you say about two lines that are perpendicular to the same line?

- Suppose you have a line *l* and a point P not lying on it. Is there a parallel to *l* through P?

- Corollary 4.1.1: Two lines perpendicular to the same line are parallel.
  - Proof: Suppose *l* and *l'* are perpendicular to *t*. By definition of perpendicular the alternate interior angels (for example) are right angles. Proposition 3.23 states that all right angles are congruent, hence by AIA theorem *l* and *l'* are parallel.
- Corollary 4.1.2: If *l* is any line and P a point not lying on it, then there exists at least one line *m* through P parallel to *l*.
  - Proof: Since P does not lie on *l* there is a line *t* perpendicular to *l* through P, by Proposition 3.16. By the same proposition there is a line *l'* through P perpendicular to *t*. Corollary 4.1.1 says that *l* and *l'* are parallel.

Converse of Alternate interior angle thm?

- Can you prove that if two lines are parallel then the alternate interior angles cut by a transversal are congruent?
  - We have found another parallel (using AIA theorem) through a point of intersection of one of the lines and a transversal. There was no reason for us to believe that those two lines are equal.
  - In fact, the converse is not a correct statement in hyperbolic plane. Next page contains some pictures for you to consider.
- Can you prove it if you had Euclidean parallel postulate?
  - The two parallels through a single point to a given line would now have to be equal, and hence the alternate interior angles would be congruent, since that is how we constructed the second parallel.



The Geometer's Sketchpad - [AlAtheorem - Half-Plane Model]
File Edit Display Construct Transform Measure Graph Window Help
angle S-U-V = 76.600°
angle T-V-U = 40.779°
t is a transversal to two parallel lines. However, the alternate interior angles are not congruent (you can see their measures above).
V
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t
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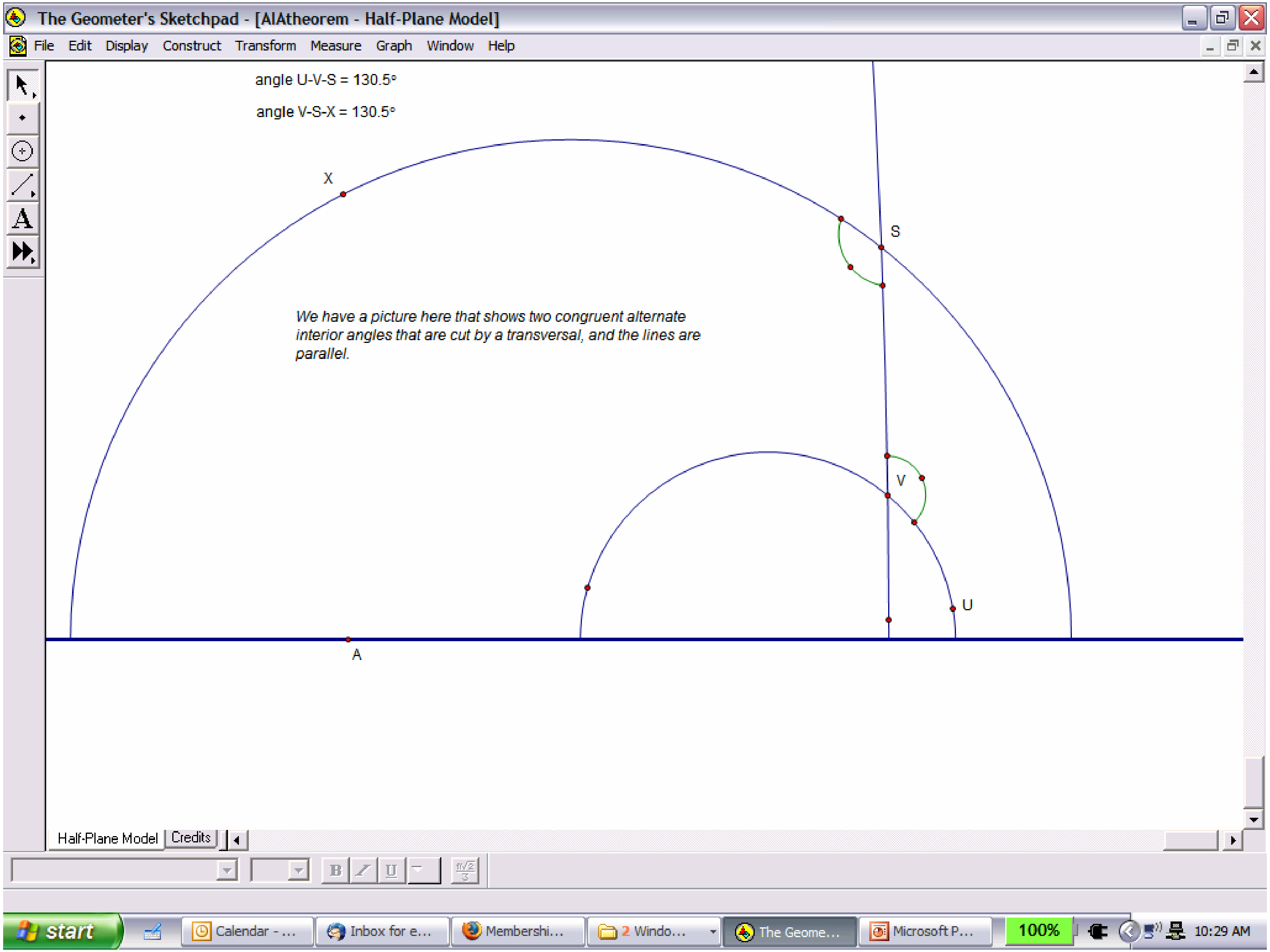
The Geometer's Sketchpad - [AIAtheorem - Half-Plane Model]
File Edit Display Construct Transform Measure Graph Window Help
angle U-V-S = 130.5°
angle V-S-X = 130.5°
X
S
We have a picture here that shows two congruent alternate interior angles that are cut by a transversal, and the lines are parallel.
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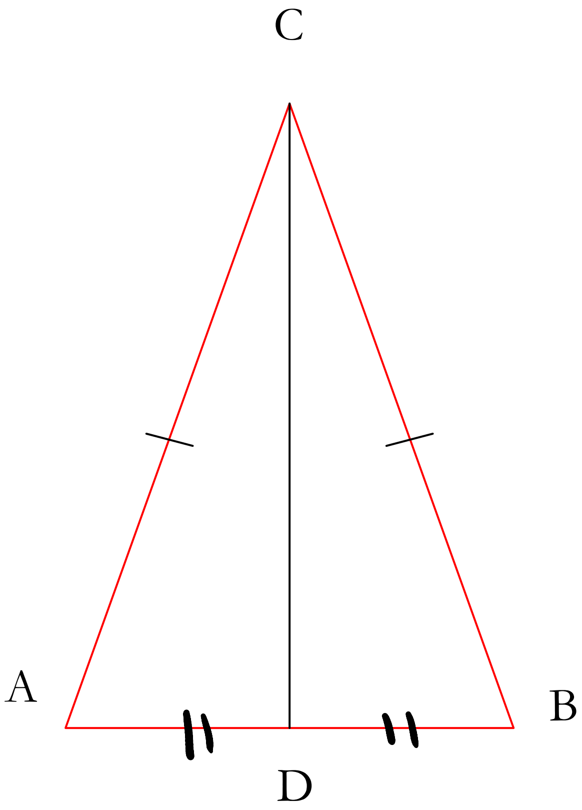
What do you think of this proof of "Base angles of isosceles triangle are congruent"?

Let ΔABC be a triangle with AC ≅ BC. Let D be a midpoint of AB. In triangles ΔACD and ΔBCD, AC ≅ BC by hypothesis. AD ≅ BD by definition of a midpoint. Therefore, triangles ΔACD and ΔBCD are congruent by SSS. Hence, ∢ A ≅ ∢ B.



To think about for Monday:

- Anything that bothers you about the proof?
  - Our comments were: we do not know what a midpoint is.
  - We agreed on definition of a midpoint:
    - A point D is a midpoint of a segment AB if A*D*B and AD ≅ DB.
    - We asked: Do midpoints exist?
  - Now it was pointed out that we don't know whether the midpoints of segments existed. Another issue that came up was: if there is a midpoint is it unique?
- What can you say about angles at D?
- What can you say about angles at C?