Class #24

Vertical angles

- Let *l* and *m* be two distinct lines meeting at B and A*B*C on *l* and D*B*E on *m*. Angles ∢ABD and ∢CBE are called *vertical angles*.

- Are angles ∢ABE and ∢CBD vertical as well? Why or why not?

Proposition 3.15

1. Vertical angles are congruent to each other.
2. An angle congruent to a right angle is a right angle.

Proof: Use Proposition 3.14.

Proof

- (a) Let ∢ABD and ∢ CBE be vertical angles, with A*B*C and D*B*E. We must show that ∢ ABD ≅ ∢ CBE. By definition of supplementary angles, ∢ ABD and ∢ ABE are supplements and ∢ CBE and ∢ ABE are supplements. By C-2, ∢ABE ≅ ∢ ABE. Therefore by Proposition 3.14, ∢ ABD ≅ ∢ CBE$.

- (b) Let ∢ ABC be a right angle. Suppose that ∢ DEF ≅ ∢ ABC. Let ∢ X be supplementary to ∢ ABC and let ∢ Y be supplementary to ∢ DEF. We must prove that ∢ DEF ≅ ∢ Y. By definition of right angle, ∢ ABC ≅ ∢ X. By Proposition3.14, ∢ X ≅ ∢ Y. We now have ∢ DEF ≅ ∢ ABC ≅ ∢ X ≅ ∢ Y. By C-2, ∢ DEF ≅ ∢ Y. Thus ∢ DEF is a right angle.

Perpendicular?

- Suppose lines *l* and *m* meet at a point A. Lines *l* and *m* are *perpendicular* if there is a point B on *l* and a point C on *m* such that $\sphericalangle BAC$ is a right angle.

- Proposition 3.16: For every line *l* and every point P there exists a line through P perpendicular to *l*.

Reminder:

- We proved: “The base angles of an isosceles triangle are congruent.”

- Equivalently: If $AB \cong AC$ in a ΔABC, then $\sphericalangle C \cong \sphericalangle B$.

- Q: If in a ΔABC you have $\sphericalangle C \cong \sphericalangle B$, what can you say about that triangle?

ASA criterion for congruence

For Monday: how you might prove these two?

Proposition 3.17: Let ΔABC and ΔDEF be two triangles. If $\sphericalangle A \cong \sphericalangle D$, $\sphericalangle B \cong \sphericalangle E$ and $AB \cong DE$, then $\Delta ABC \cong \Delta DEF$.

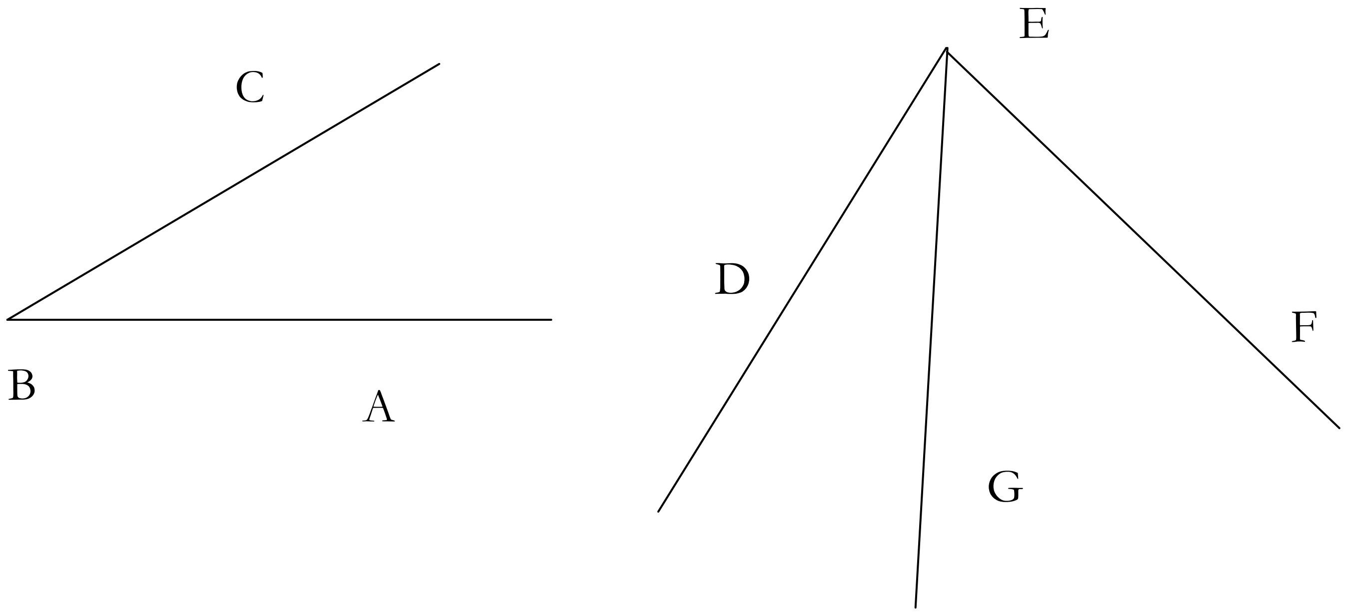
Proposition 3.18: If in a ΔABC we have $\sphericalangle A \cong \sphericalangle B$, then ΔABC is isosceles.

Angle addition, subtraction and comparison

- There are equivalent statements for congruence of angles to those we were either given as axioms or that we proved for segments. Here is a sampling:
- Proposition 3.19 (Angle addition): If $\overrightarrow{BG}$ is between $\overrightarrow{BA}$ and $\overrightarrow{BC}$, $\overrightarrow{EH}$ is between $\overrightarrow{ED}$ and $\overrightarrow{EF}$, $\sphericalangle CBG \cong \sphericalangle FEH$, and $\sphericalangle GBA \cong \sphericalangle HED$, then $\sphericalangle ABC \cong \sphericalangle DEF$.
- Proposition 3.20 (Angle Subtraction): If $\overrightarrow{BG}$ is between $\overrightarrow{BA}$ and $\overrightarrow{BC}$, $\overrightarrow{EH}$ is between $\overrightarrow{ED}$ and EF, $\sphericalangle CBG \cong \sphericalangle FEH$, and $\sphericalangle ABC \cong \sphericalangle DEF$, then $\sphericalangle GBA \cong \sphericalangle HED$.

Comparison

- $\sphericalangle ABC < \sphericalangle DEF$ means there is a ray EG between ED and EF such that $\sphericalangle ABC \cong \sphericalangle DEG$.



Proposition 3.21 (Ordering of angles)

1. Exactly one of the following happens: $\sphericalangle P < \sphericalangle Q$, $\sphericalangle P \cong \sphericalangle Q$, or $\sphericalangle Q < \sphericalangle P$.
2. If $\sphericalangle P < \sphericalangle Q$ and $\sphericalangle Q \cong \sphericalangle R$, then $\sphericalangle P < \sphericalangle R$.
3. If $\sphericalangle P < \sphericalangle Q$ and $\sphericalangle P \cong \sphericalangle R$, then $\sphericalangle R < \sphericalangle Q$.
4. If $\sphericalangle P < \sphericalangle Q$ and $\sphericalangle Q < \sphericalangle R$, then $\sphericalangle P < \sphericalangle R$.

To think about for Monday!!!

- Proposition 3.22 (SSS): Let ΔABC and ΔDEF be two triangles. If $AB \cong DE$, $BC \cong EF$ and $AC \cong DF$, then $\Delta ABC \cong \Delta DEF$.

- Proposition 3.23: All right angles are congruent.

Hints are to be found in the book.