Class #20

Axiom of congruence

Changing Pasch's theorem

- *Pasch's theorem:* If A, B, C are distinct, noncollinear points and *l* is any line intersecting AB in a point between A and B, then *l* also intersects AC or BC. If C does not lie on *l*, then *l* does not intersect both AC and BC.

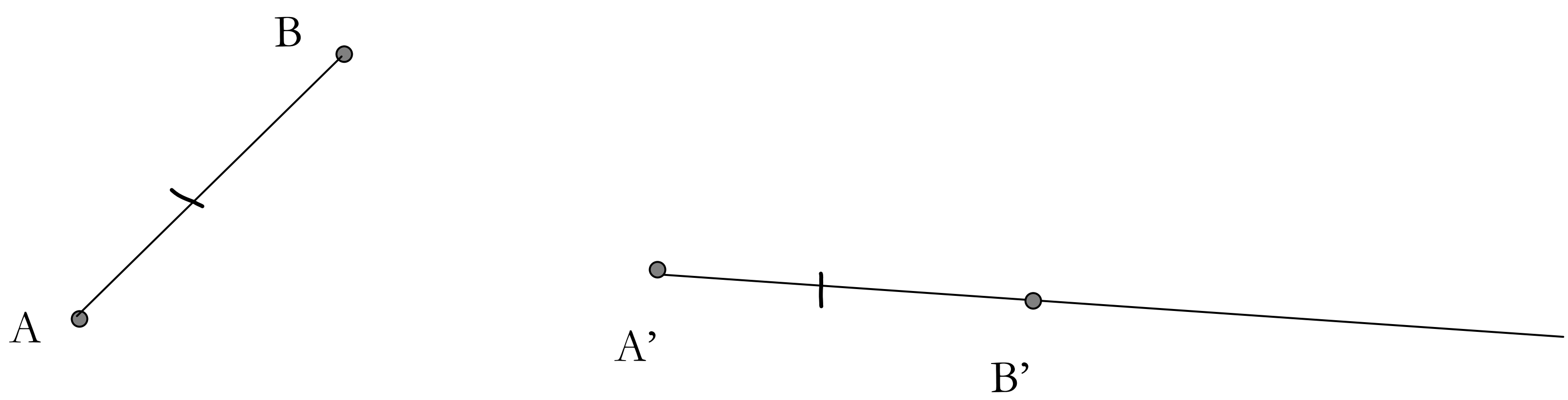
- Can you come up with a theorem in spirit of Pasch's if a line *l* is replaced by a ray *r*.

Congruence!!!

- Last undefined term
- What do you think of when you hear the word congruent?
- We'll really be thinking of two different things:
  - Congruent segments
  - Congruent angles
- Define: Two triangles are congruent if there is a one to one correspondence between their vertices so that corresponding sides are congruent and so that corresponding angles are congruent.
- If we write $\Delta ABC \cong \Delta EFG$ we will mean that E corresponds to A, F corresponds to B and G corresponds to C.

C1 – Congruence Axiom 1

- If A and B are distinct points and if A' is any point, then for each ray *r* emanating from A', there is a unique point B' on *r*, such that B' ≠ A' and AB ≅ A'B'.



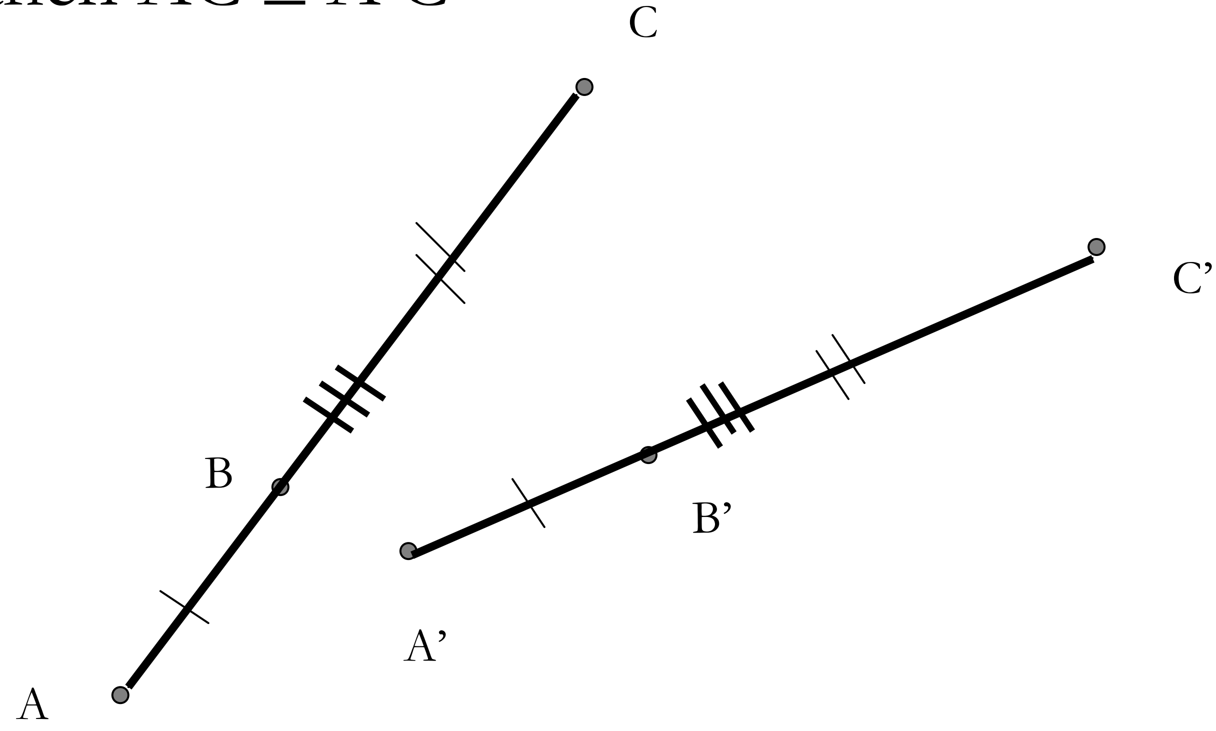
C2 – Congruence Axiom 2

- If AB ≅ CD and AB ≅ EF then CD ≅ EF. Furthermore, every segment is congruent to itself.

- Lemma 3.9.5: Congruence of segments is equivalence relation.
  - AB ≅ AB, by ***C2*** -- reflexive
  - Since AB ≅ CD and AB ≅ AB, by ***C2***, we see CD ≅ AB – symmetric
  - Transitive by ***C2.***

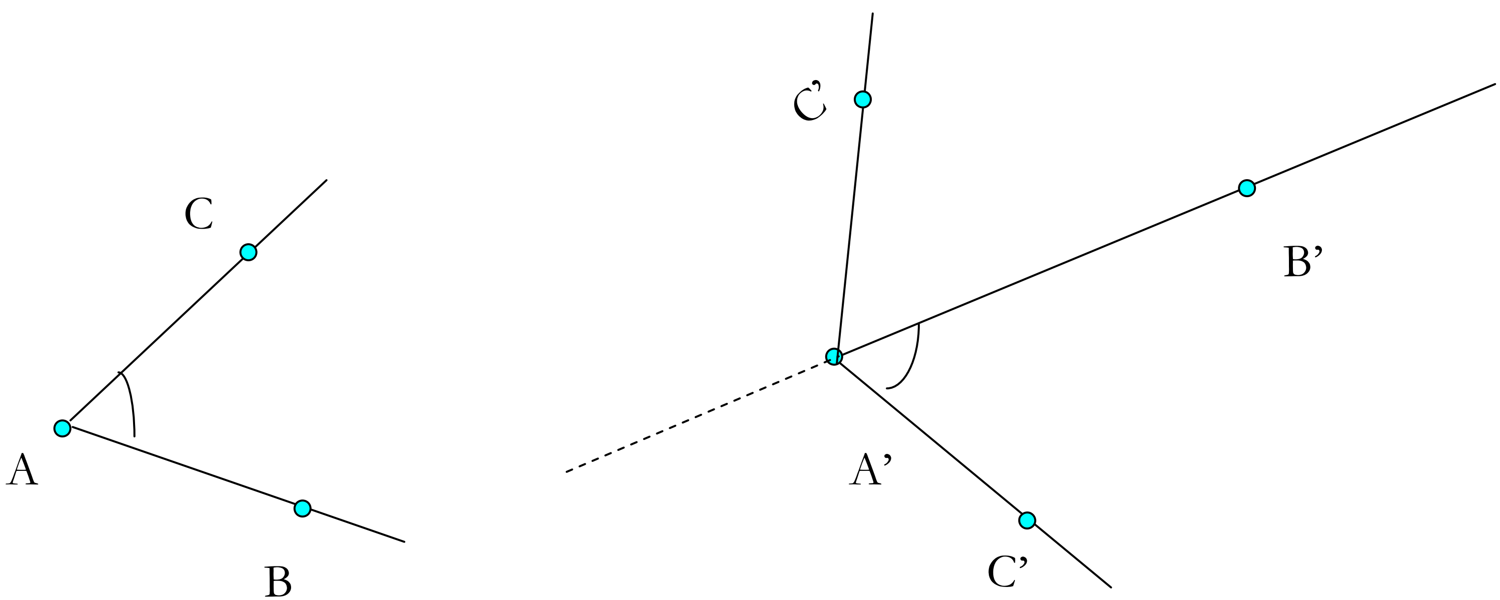
C3 – Congruence Axiom 3

- If A*B*C, A'*B'*C', AB ≅ A'B' and BC ≅ B'C', then AC ≅ A'C'



C4 – Congruence Axiom 4

- Given any ∢BAC and given any ray A’B’ there is a unique ray A’C’ on a given side of the line A’B’ such that ∢BAC ≅ ∢B’A’C’.

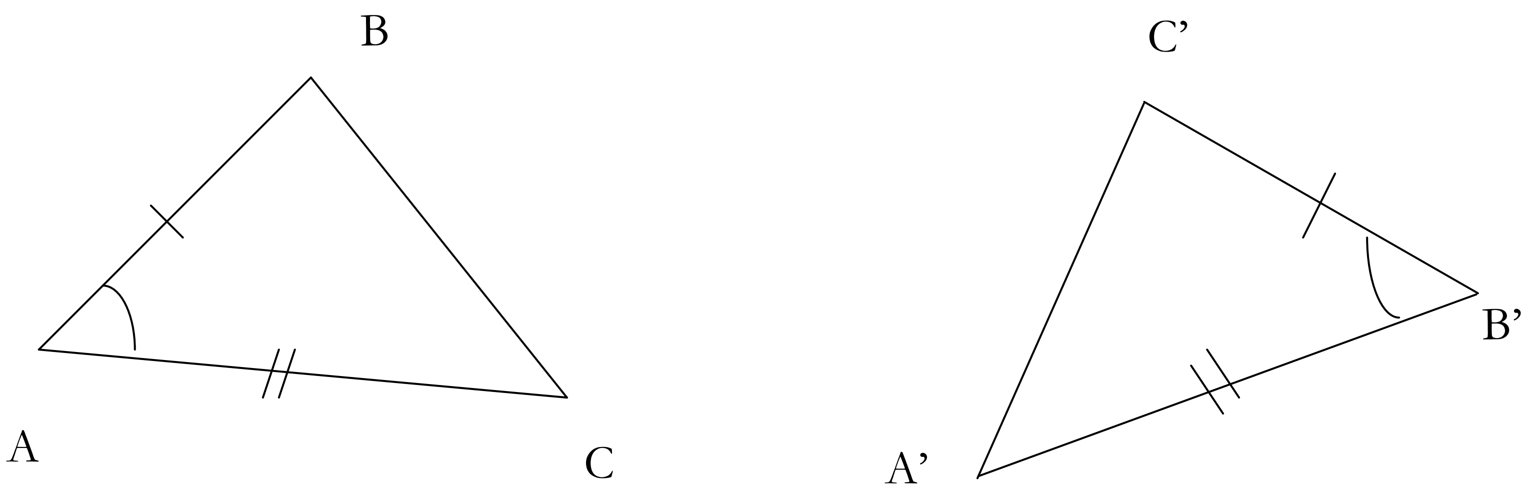


C5 – Congruence Axiom 5

- If $\sphericalangle A \cong \sphericalangle B$ and $\sphericalangle B \cong \sphericalangle C$ then $\sphericalangle A \cong \sphericalangle C$. Moreover, every angle is congruent to itself.
- Lemma 3.9.6: Congruence of angles is equivalence relation.

C6 (SAS) – Congruence Axiom 6

- If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the two triangles are congruent.



Δ ABC $\cong$ Δ B'C'A'

Δ BAC $\cong$ Δ C'B'A'