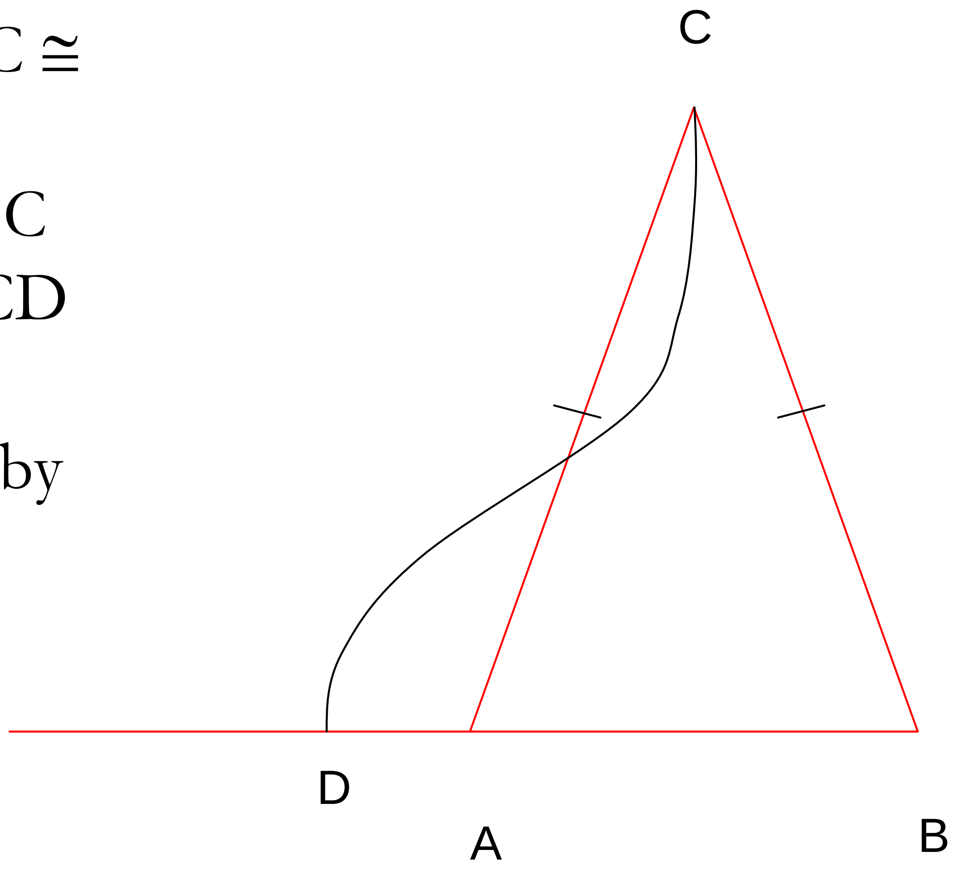
Class #18

Pasch's theorem

Base angles of isosceles triangle are congruent

Let ABC be a triangle with AC ≅ BC. By Theorem X, ∢C has a bisector. Let the bisector of ∢C meet AB at D. In triangles ACD and BCD, AC ≅ BC by hypothesis. ∢ACD ≅ ∢BCD, by definition of a bisector. Therefore, triangles ACD and BCD are congruent by SAS. Hence, ∢A ≅ ∢B.



Questions:

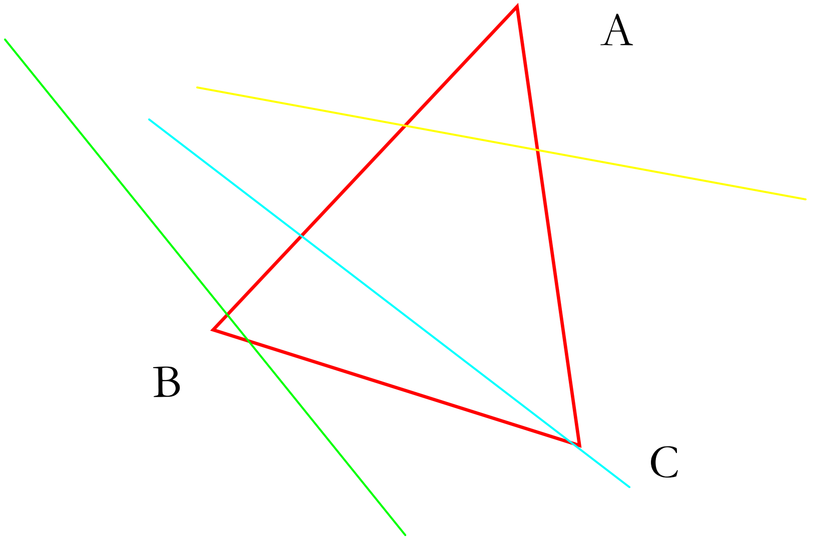
- Triangle?
- Isosceles?
- Base?
- Angle?
- Congruent?

Triangle

- If A, B and C are three distinct noncollinear points, the *triangle* Δ ABC is the union of segments AB, BC and AC. Points A, B and C are called vertices of the triangle, and segments AB, AC and BC are called sides.

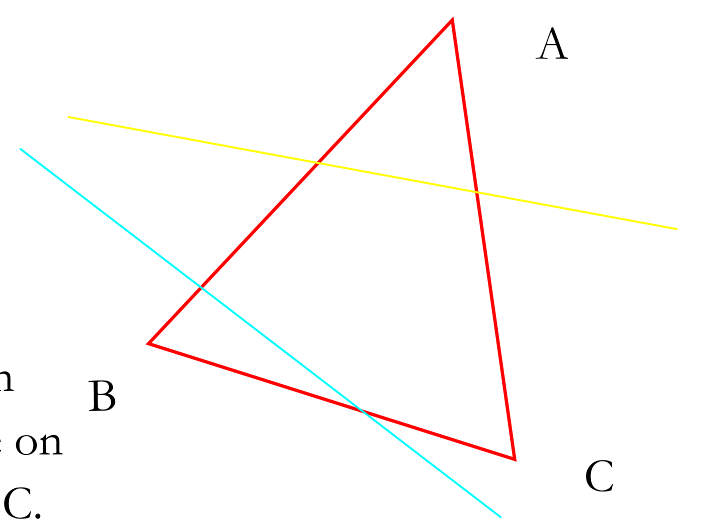
Pasch's theorem

- If A, B, C are distinct, noncollinear points and *l* is any line intersecting AB in a point between A and B, then *l* also intersects AC or BC. If C does not lie on *l*, then *l* does not intersect both AC and BC.
  - Is the wording sloppy?
  - Is there redundancy?



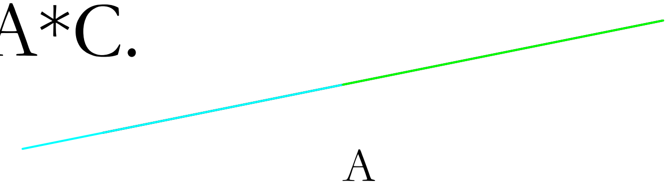
Sketch of the proof of Pasch's Theorem

- Let A, B, C be three distinct, noncollinear points and *l* a line intersecting AB in a point between A and B. Then one of the following happens:
  - C lies on *l*. Then *l* intersects both BC and AC.
  - C does not lie on *l*. Consider the points A and C. Either
    - they lie on the same side of *l*. In this case *l* does not intersect AC. However together with A&B on opposite sides of *l*, and B4, we conclude that B&C are on opposite sides of *l*, hence *l* intersects BC.
    - they lie on opposite sides of *l* which means that AC intersects *l*. Further, since A&B are on opposite sides of *l*, we conclude that B&C are on the same side of *l*, hence *l* does not intersect BC.

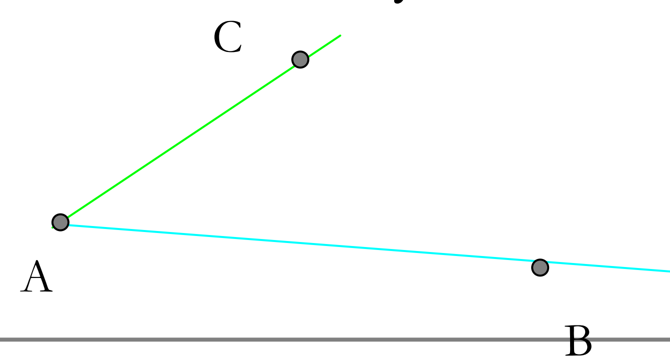


Angle?

- Given three distinct points A, B and C, the rays $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are called *opposite* if B*A*C.



- An *angle* with vertex A is a point A together with two distinct nonopposite rays $\overrightarrow{AB}$ and $\overrightarrow{AC}$, denoted by $\measuredangle CAB$ and $\measuredangle BAC$. These two rays are called the *sides* of the angle.

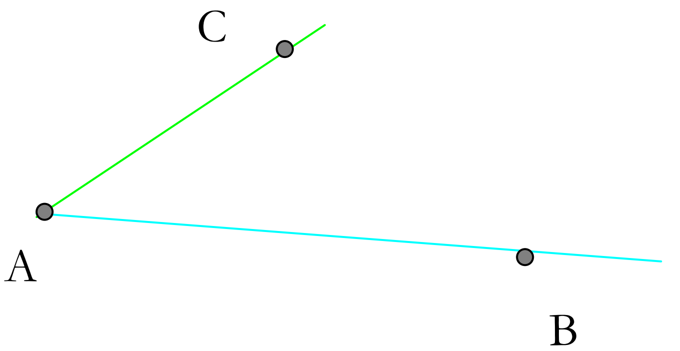


To ask:

- Is it a good definition?
- Is it precise?
- If E is a point on ray AB, is $\angle$EAC same as $\angle$ BAC? Do you want it to be?

  It is the same due to:

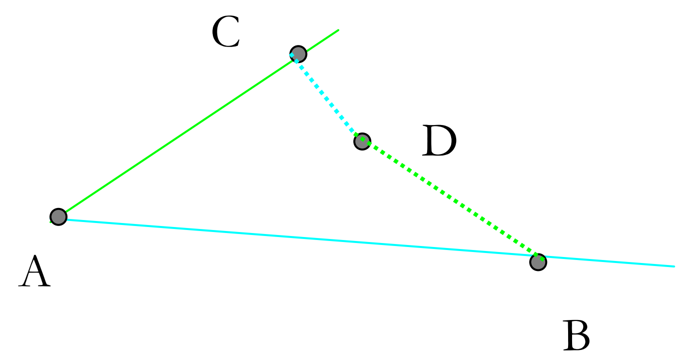
  - Proposition 3.6: If A*B*C, then B is the only point in common to rays BA and BC, and AB=AC



- Does the definition correspond to what you think of an angle?

Interior of an angle?

- A point D is in the *interior* of an angle $\measuredangle$ BAC if D is on the same side of $\overleftrightarrow{AC}$ as B and on the same side of $\overleftrightarrow{AB}$ as C.



- Equivalently,

  int $\measuredangle$ BAC = side(B, $\overleftrightarrow{AC}$) $\cap$ side(C, $\overleftrightarrow{AB}$)

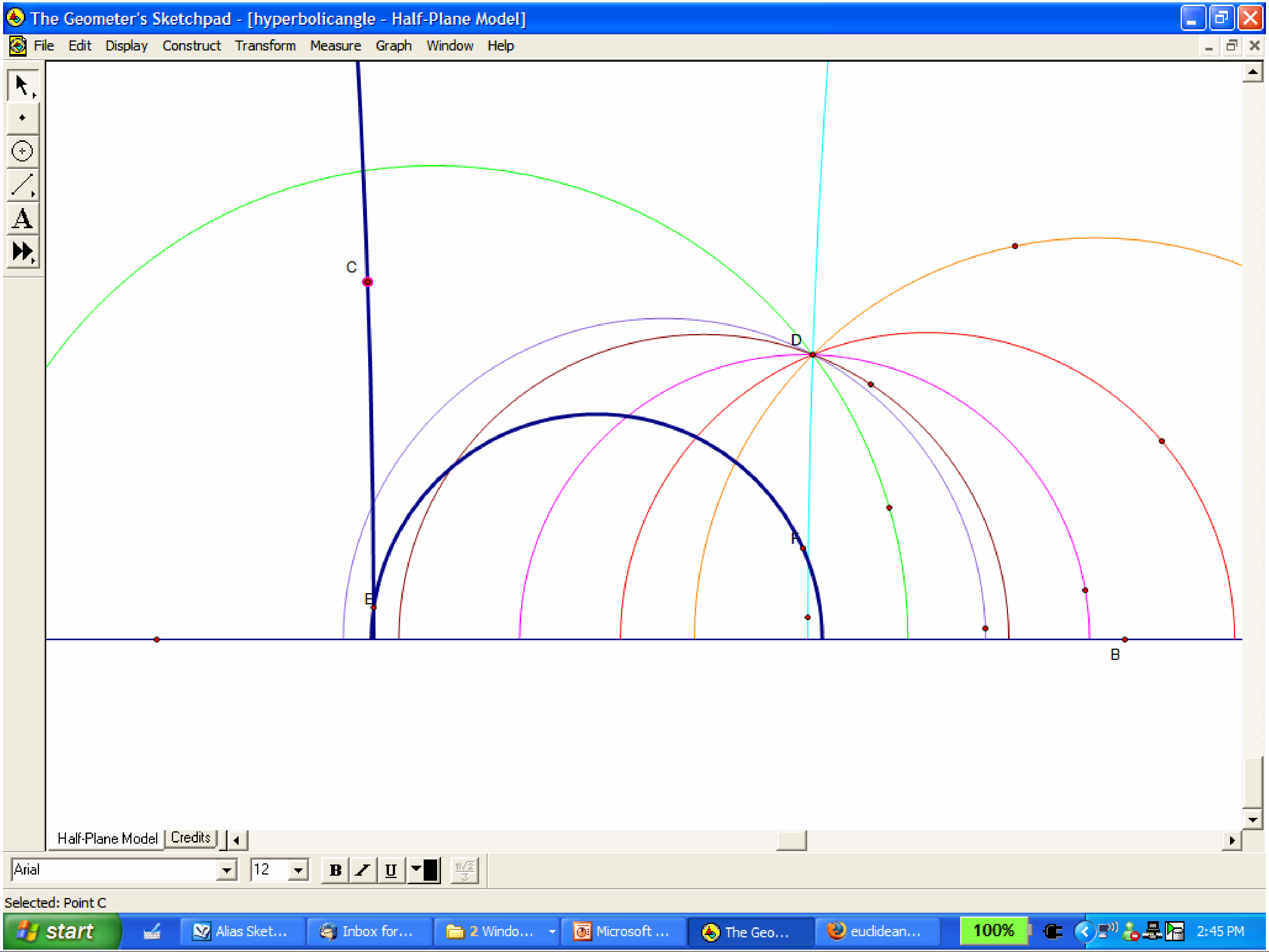
Proposition 3.7: Let $\angle BAC$ be an angle and D any point lying on $\overleftrightarrow{BC}$. D is in the interior of $\angle BAC$ iff B*D*C.

- Proof: Let $\measuredangle BAC$ be an angle and D any point lying on BC.
  - ⇒ If D is in the interior of $\measuredangle BAC$ then B and D are on the same side of $\overleftrightarrow{AC}$, hence BD does not intersect $\overleftrightarrow{AC}$. We know that the line $\overleftrightarrow{BD}$ does intersect $\overleftrightarrow{AC}$, hence D lies between the point of intersection, which is C, and B.
  - ⇐ If B*D*C, then BD does not intersect $\overleftrightarrow{AC}$, hence B and D are on the same side of $\overleftrightarrow{AC}$. Similarly, DC does not intersect $\overleftrightarrow{AB}$, so D and C are on the same side of $\overleftrightarrow{AB}$. By definition, D lies in the interior of $\measuredangle BAC$.

Not a fact!

- It is not true that if D is in the interior of $\angle BAC$ that D then lies on a segment "connecting" two sides of the angle.

See the following picture.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