Class #16

Towards LSP

Proposition 3.2: Every line has exactly two sides and they are disjoint.

- This proof will consist of three parts.

1. There are at least two sides
2. There are no more than two sides
3. These two sides are disjoint

1. There are at least two sides

- Let l be a line. By Proposition 2.3 there is a point P not on l. By ***I-2***, there is a point R on l. By ***B-2***, there is a point Q such that P*R*Q. By definition P and Q are on opposite sides of l. We have

$$P \notin \mathrm{side}(Q, l) \text{ and } Q \notin \mathrm{side}(P, l)$$

In particular, $\mathrm{side}(Q, l) \neq \mathrm{side}(P, l)$.

2. There are no more than two sides

Suppose S is not a point lying on l. Then S and P are either on the same side of l or on opposite sides. If S and P are on the same side of l, then $S \in$ side(P, l). By Lemma 3.1.5 side(S, l) = side(P, l).

If S and P are on opposite sides, then by ***B-4***, S and Q are on the same side of l, so $S \in$ side(Q, l). We now have side(S, l) = side(Q, l), by Lemma 3.1.5. Therefore, side(P,l) and side(Q, l) are the only two sides.

3. These two sides are disjoint

We want to show that side(P, l) $\cap$ side(Q, l) = $\emptyset$.

Suppose not, i.e. there is a point S$\in$ side(P, l) $\cap$ side(Q, l) . Since S$\in$ side(P, l) we have that S and P are on the same side of l. Since S$\in$ side(Q, l) we have that S and Q are on the same side of l. By B-4, P and Q are on the same side of l, contradiction.

Question:

- If side(P, l) $\cap$ side(Q, m)=$\emptyset$, what can you say about P, Q, l and m?
  - $l = m$ and P and Q are on opposite sides of l
  - $l \parallel m$. Let L be the point on l that lies on PQ (exists because P and Q are on opposite sides of l), so P*L*Q. Let M be the point on m that lies on PQ (exists because P and Q are on opposite sides of m), so P*M*Q.
    - We can conclude that P*L*M and L*M*Q. Why? Is this true in general?