Class #15

Don't just read it; fight it! Ask your own questions, look for your own examples, discover your own proofs. Is the hypothesis necessary? Is the converse true? What happens in the classical special case? What about the degenerate cases? Where does the proof use the hypothesis?

Paul R. Halmos, I Want to be a Mathematician

Question

- Is the real projective plane still a model (I mean of ***I1-3*** and ***B1-3***)?
- What would A*B*C mean?
- Not a model of incidence and betweenness: check out the Appendix A in the book.

Definition

- Let l be any line and A, B points not lying on l.
  - If A=B or AB contains no point lying on l, then say that A and B are on the same side of l.
  - If A≠B and AB intersects l, then say that A and B are on opposite sides of l.

- Remark:
  - "AB contains no point lying on l" means $AB \cap \{l\} = \varnothing$
  - "AB intersects l" means $AB \cap \{l\} \neq \varnothing$

Plane separation axiom (PSP)

B-4: For any line *l* and any three points A, B and C not lying on *l* :

1. If A and B are on the same side of *l* and B and C are on the same side of *l* then A and C are on the same side of *l*.
2. If A and B are on opposite sides of *l*, and B and C are on opposite sides of *l*, then A and C are on the same side of *l*.

Corollary: If A and B are on opposite sides of *l* and B and C are on the same side of *l*, then A and C are on opposite sides of *l*.

Claim: PSP does not follow from LSP and earlier claims.

Model for *LSP* and *I1-3* and *B1-3* where *PSP* is incorrect

- Points: triples of real numbers **p**=(x, y, z)
- Lines: equations of the form **p**=**v** t+**a**
- Lies on: satisfies the equation
- Between: **a*****c*****b** if **c**=(1-t)**a**+t**b**, $0 \leq t \leq 1$

What needs to be done?

- Show that the axioms and *LSP* are satisfied
- Show that *PSP* is not satisfied.

- Given a line l and a point P not on l, the *side* of l containing P is the set

side(P, l)={Q | P and Q are on the same side of l}

- side(P,l) is also called a *half-plane* bound by l.

Exercise: Prove

Lemma 3.1.5: If A and B are on the same side of l, then

$$\text{side}(A, l) = \text{side}(B, l)$$

Proof:

To show that side(A, l)= side(B, l) we need to show two things:

1. side(A, l) $\subseteq$ side(B, l)
2. side(B, l)$\subseteq$ side(A, l).

1. If C$\in$ side(A, l), then by definition A and C are on the same side of l. Since A and B are on the same side of l, by ***B-4(i)***, B and C are on the same side of l. hence C$\in$ side(B, l). This shows that side(A, l) $\subseteq$ side(B, l).
2. The proof that side(B, l)$\subseteq$ side(A, l) would proceed in exactly the same way as the one above with roles of side(B, l) and side(A, l) reversed.

Is the converse true?

- What is the converse?

Converse of Lemma 3.1.5:

If side(A, *l*)= side(B, *l*) then A and B are on the same side of *l*.

- Proof:

It remains to be shown that if side(A, *l*)= side(B, *l*) then A and B are on the same side of *l*. Let Q be a point on the same side of *l* as A. By definition, Q ∈ side(A, *l*). Since side(A, *l*)= side(B, *l*), Q belongs to side(B, *l*) as well, hence Q and B are on the same side of *l*. By ***B4(i)***, A and B are on the same side of *l*.

Better Lemma 3.1.5

Lemma 3.1.5. Points A and B are on the same side of line l if and only if side(A, l) = side(B, l).