Class #14

LSP, ***B-4***, ……

Line Separation Property (LSP): If C*A*B and *l* is a line through A, B and C then for every point P on *l* either $P \in \overrightarrow{AB}$ or $P \in \overrightarrow{AC}$.

Claim: LSP is independent of ***I1-3*** & ***B1-3.***

Independence of *LSP*

- *LSP* holds in the Cartesian plane where

B*A*C if $A = B + (C - B)\,t$, where $0 < t < 1$.

  - Do you understand what this is saying? Do a few examples.
  - You may wish to check that this in fact corresponds to our notion of betweenness on the lines. Do you see why it works?
  - You may consider how this definition compares to the one given in class in terms of elegance, clarity, efficiency, and so forth.

- *LSP* does not hold in the following model:

Let A=(0,0), P=(1,0), B=(2,0)

The points, lines and lies on is the same as in the Cartesian plane. We will define new "between" relation

X♣Y♣Z if X*Y*Z when (X≠P and Y ≠A and Z ≠ B)

and

P♣A♣B

  - Are all the axioms satisfied?

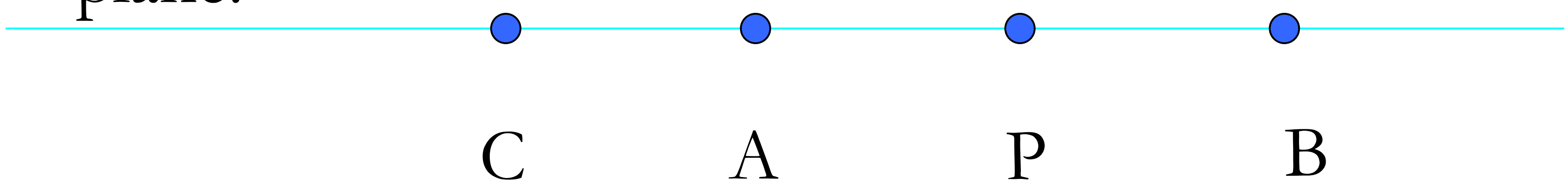
Check that *LSP* does not hold:

- Point C=(-1,0) lies $l = \overleftrightarrow{AB}$, and we will consider rays $\overrightarrow{AP}$ and $\overrightarrow{AC}$.

- Point B certainly lies on *l*.

- B does not lie on $\overrightarrow{AP}$, since P ♣ A ♣ B
- B does not lie on $\overrightarrow{AC}$, since C ♣ A ♣ B
  - Write down the definition of each ray but using ♣, not *!!!

- Here is a "picture", but this is in the Cartesian plane:



$B \notin \overrightarrow{AP} = \{A, P\} \cup \{X: A \clubsuit X \clubsuit P\} \cup \{Y: A \clubsuit P \clubsuit Y\}$ because $P \clubsuit A \clubsuit B$

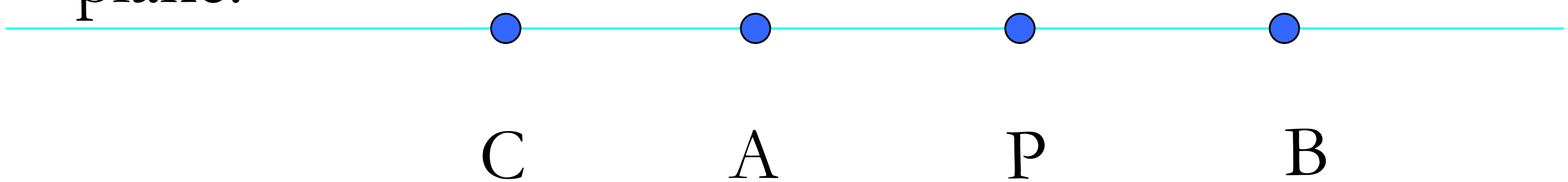
$B \notin \overrightarrow{AC} = \{A, C\} \cup \{X: C \clubsuit X \clubsuit A\} \cup \{Y: Y \clubsuit C \clubsuit A\}$ because $C \clubsuit A \clubsuit B$

Another scenario:

- Point C=(-1,0) lies $l = \overleftrightarrow{AB}$, and we will consider rays $\overrightarrow{AP}$ and $\overrightarrow{AB}$.

- C does not lie on $\overrightarrow{AP}$, since C ♣ A ♣ P
- C does not lie on $\overrightarrow{AB}$, since C ♣ A ♣ B

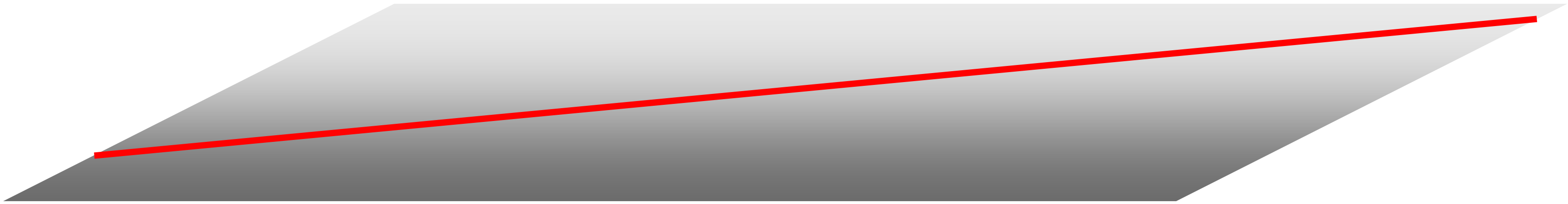
- Here is another "picture", again in the Cartesian plane:



$C \notin \overrightarrow{AP} = \{A, P\} \cup \{X\text{: } A\clubsuit X\clubsuit P\} \cup \{Y\text{: } A\clubsuit P\clubsuit Y\}$ because $C\clubsuit A\clubsuit P$

$C \notin \overrightarrow{AB} = \{A, B\} \cup \{X\text{: } C\clubsuit X\clubsuit A\} \cup \{Y\text{: } Y\clubsuit C\clubsuit A\}$ because $C\clubsuit A\clubsuit B$

- We conclude that we need another axiom. We could add *LSP* or *B4P*, but it turns out that if we wanted a complete system, this axiom alone would not be enough. In order to add as few axioms as possible, we will add something completely different and show *LSP* and *B4P* from it.



What would you say the red line does to the plane?

Try to define this without ever using a word plane. You may need other terms before you get to that. Think about what it is that you are trying to describe and think about the terms you know so far.