Class #13

More betweenness

Proposition 3.1: For any two distinct points A and B:

1. $\overrightarrow{AB} \cap \overrightarrow{BA} = AB$ (overlapping rays)
2. $\overrightarrow{AB} \cup \overrightarrow{BA} = \{ \overleftrightarrow{AB} \}$

Prove 1. We will leave 2. for homework

Proof of Proposition 3.1 (1.)

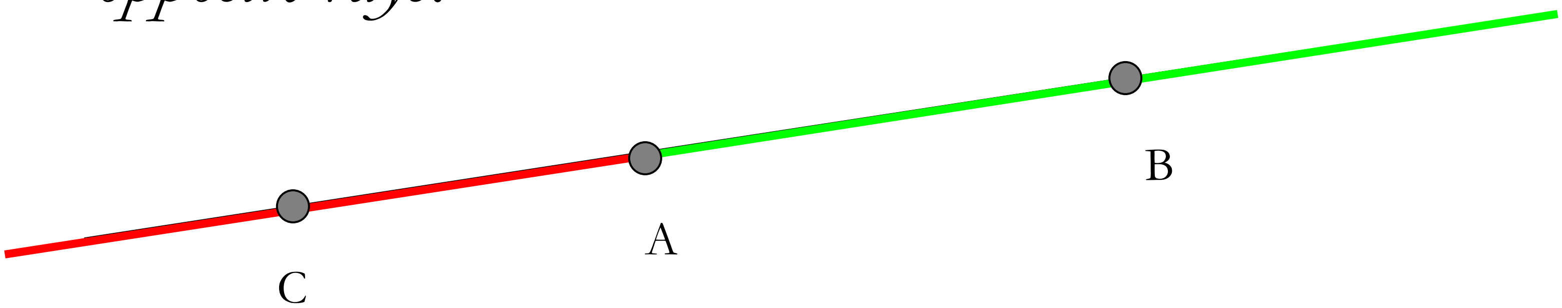
We need to show two things:

1. $AB \subset \overrightarrow{AB} \cap \overrightarrow{BA}$
2. $\overrightarrow{AB} \cap \overrightarrow{BA} \subset AB$

1. That $AB \subset \overrightarrow{AB} \cap \overrightarrow{BA}$ means that $AB \subset \overrightarrow{AB}$ and $AB \subset \overrightarrow{BA}$. By definition of a ray $AB \subset \overrightarrow{AB}$. By Lemma 3.0(1) AB=BA, and by definition of a ray $BA \subset \overrightarrow{BA}$, so $AB \subset \overrightarrow{BA}$. Hence $AB \subset \overrightarrow{AB} \cap \overrightarrow{BA}$
2. We now show that $\overrightarrow{AB} \cap \overrightarrow{BA} \subset AB$. Let $P \in \overrightarrow{AB} \cap \overrightarrow{BA}$. If P=A or P=B, then $P \in AB$ by definition of a segment. Suppose that P, A and B are distinct points. Note that they are collinear: $P \in \overrightarrow{AB} \cap \overrightarrow{BA} \Rightarrow P \in \overrightarrow{AB} \Rightarrow P \in AB$ or A*B*P ⇒ A*P*B or A*B*P, hence by ***B-1*** these three points are collinear. Since A, B, P are three distinct points, by ***B-3***, exactly one of the following holds: A*P*B or A*B*P or P*A*B.

- If A*B*P, then by ***B-3***, not(A*P*B) and not(P*A*B). Since not(A*P*B) then by ***B-1***, not(B*P*A), which by definition of a segment means that $P \notin BA$. Also, not(P*A*B) means not(B*A*P). By definition of a ray, $P \notin BA$ and not(B*A*P) show that $P \notin \overrightarrow{BA}$, which contradicts our assumption.
- If P*A*B, similar argument shows that $P \notin \overrightarrow{AB}$, which is a contradiction.
- If A*P*B, then by definition of a segment means that $P \in AB$.

- Definition: If C*A*B, then $\overrightarrow{AC}$ and $\overrightarrow{AB}$ are called *opposite rays.*



Obvious claim?

- If C*A*B and l is a line through A, B and C then for every point P on l either $P \in \overrightarrow{AB}$ or $P \in \overrightarrow{AC}$.

Exercise:

- Prove the obvious claim:

If $C*A*B$ and l is a line through A, B and C then for every point P on l either $P \in \overrightarrow{AB}$ or $P \in \overrightarrow{AC}$.

Try #1:

- If P=A or P=C, we're done. Else P, A and C are distinct, so by ***B-3***, one of the following holds:

- P*A*C → $P \in \overrightarrow{CA}$???

  A*P*C → $P \in \overrightarrow{AC}$

  A*C*P → $P \in \overrightarrow{AC}$

Try #2:

- If P is A, B or C, we're done.
- P, A and B are distinct, so by ***B-3***, one of the following holds:

1. P*A*B 2. A*P*B 3. A*B*P

- P, A and C are distinct, so by ***B-3***, one of the following holds:

4. P*A*C 5. A*P*C 6. A*C*P

If 2., 3., 5. or 6. done.

If 1. or 4. P*A*B and P*A*C. So?

Line Separation Property (LSP): If C*A*B and l is a line through A, B and C then for every point P on l either $P \in \overrightarrow{AB}$ or $P \in \overrightarrow{AC}$.

Equivalently: If C*A*B and l is the line through A, B and C then $\{l\} = \overrightarrow{AB} \cup \overrightarrow{AC}$.

LSP is related to another "obvious" claim:
(B4P): If A*B*C and A*C*D then B*C*D and A*B*D.

Homework for WEDNESDAY!

Show that *LSP* is independent of ***I1-3*** and ***B1-3.***