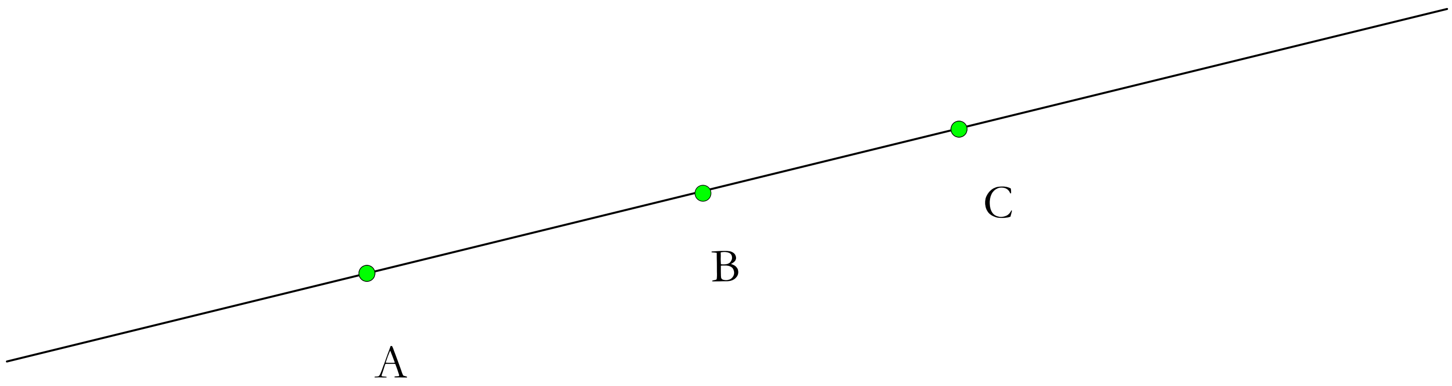
Class #12

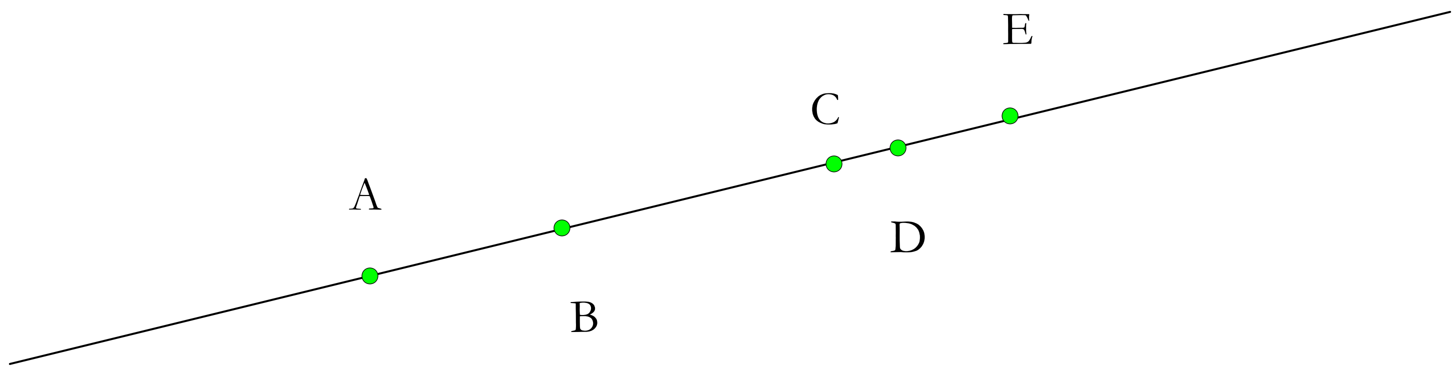
Betweenness

Betweenness axioms

- Would you say that one of these points is between the other two?

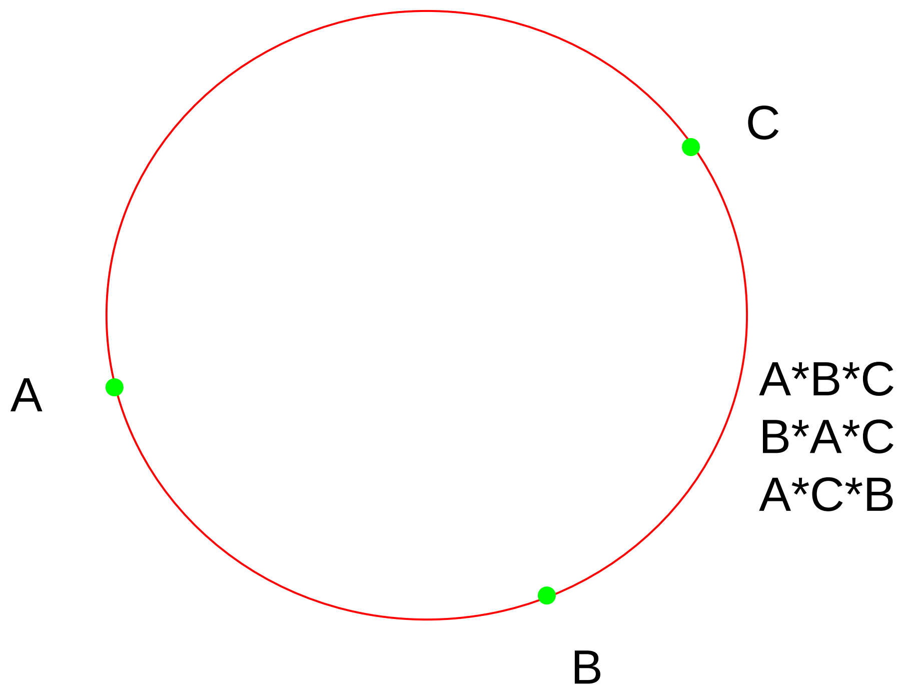


- ***B-1***: If A*B*C, then A, B and C are three distinct points lying on the same line and C*B*A.



B-2: For any two distinct points B and D, there exist points A, C, and E on $\overleftrightarrow{BD}$ such that A*B*D, B*C*D and B*D*E.

B-3: If A, B and C are three distinct points lying on the same line then one and only one of the points is between the other two.



Food for thought:

- What have we gained by adding these axioms?
- Did we lose anything? Think about models.
- Define segment and ray.

Definitions

- Given two distinct points A and B, the *segment* AB is the set of all points between A and B, together with A and B.
  - AB = {C: A*C*B} ∪ {A, B}
- Given two distinct points A and B, the *ray* $\overrightarrow{AB}$ is the set of all points on the segment AB together with all the points C such that A*B*C.
  - $\overrightarrow{AB} = AB \cup \{C: A*B*C\}$

Lemma 3.0: For any two distinct points A, B:

1. $AB = BA$
2. $AB \subsetneq \overrightarrow{AB}$

Q: How do you show that two sets are equal?

To show that $S \subset T$, you have to show that every element of S is also element of T: if x in S then x in T. To show that $S = T$, you have to show that $S \subset T$ and $T \subset S$.

Proof of 3.0.

- We first show that AB ⊂ BA. Let T be a point in AB. By definition of a segment A*T*B or T=A or T=B. If T=A or T=B, then by definition of a segment T∈ BA. If A*T*B, then by ***B-1*** B*T*A, hence by definition of a segment T∈ BA.

  Repeat the argument with the roles of AB and BA reversed to conclude that AB=BA.

- If T∈ AB, then T∈ $\overrightarrow{AB}$, by definition of a ray. We need to show that AB ≠ $\overrightarrow{AB}$, which means we need to find a point in $\overrightarrow{AB}$ which is not in AB. By axiom ***B-2***, there exists a point E such that A*B*E. By definition of a ray E ∈ $\overrightarrow{AB}$. By ***B-1*** A, B and E are distinct points, so E ≠ A and E ≠ B. Further, by ***B-3***, not(A*E*B), thus E ∉ AB.