Class #11

Finish up models and move on

Modified Model#2

- For each set of parallel lines add a new point to the model#2 that will lie on each of those parallel lines. If a line does not have any parallels then add to the model#2 a new point that will lie on that line only.
  - Points: A, B, C, D, E, F, G
  - Lines: {A,B,E}, {C,D,E}, {A,C,F}, {B,D,F}, {A,D,G}, {B,C,G}, {E,F,G}
- Which parallel postulate holds in this new model?

Relation ~

- Let $\mathcal{A}$ be an affine plane. Define

$$l \sim m \text{ if } (l = m \text{ or } l \parallel m)$$

This relation is

  - *reflexive* $(l \sim l)$
  - *symmetric* $(l \sim m \Rightarrow m \sim l)$
  - *transitive* $((l \sim m \text{ and } m \sim n) \Rightarrow l \sim n)$,

- Every relation that has the above properties is called *equivalence relation*.

Equivalence classes

- $[l] = \{\text{all lines } m \text{ such that } l \sim m\}$
- If $l \sim m$, then $[l]=[m]$
- My recipe:
  - "For each set of parallel lines add a new point to the model#2 that will lie on each of those parallel lines. If a line does not have any parallels then add to the model#2 a new point that will lie on that line only. "

can be restated as follows:

  - "For each equivalence class $[l]$, add a point $P_{[l]}$ which will lie on each line in the equivalence class"

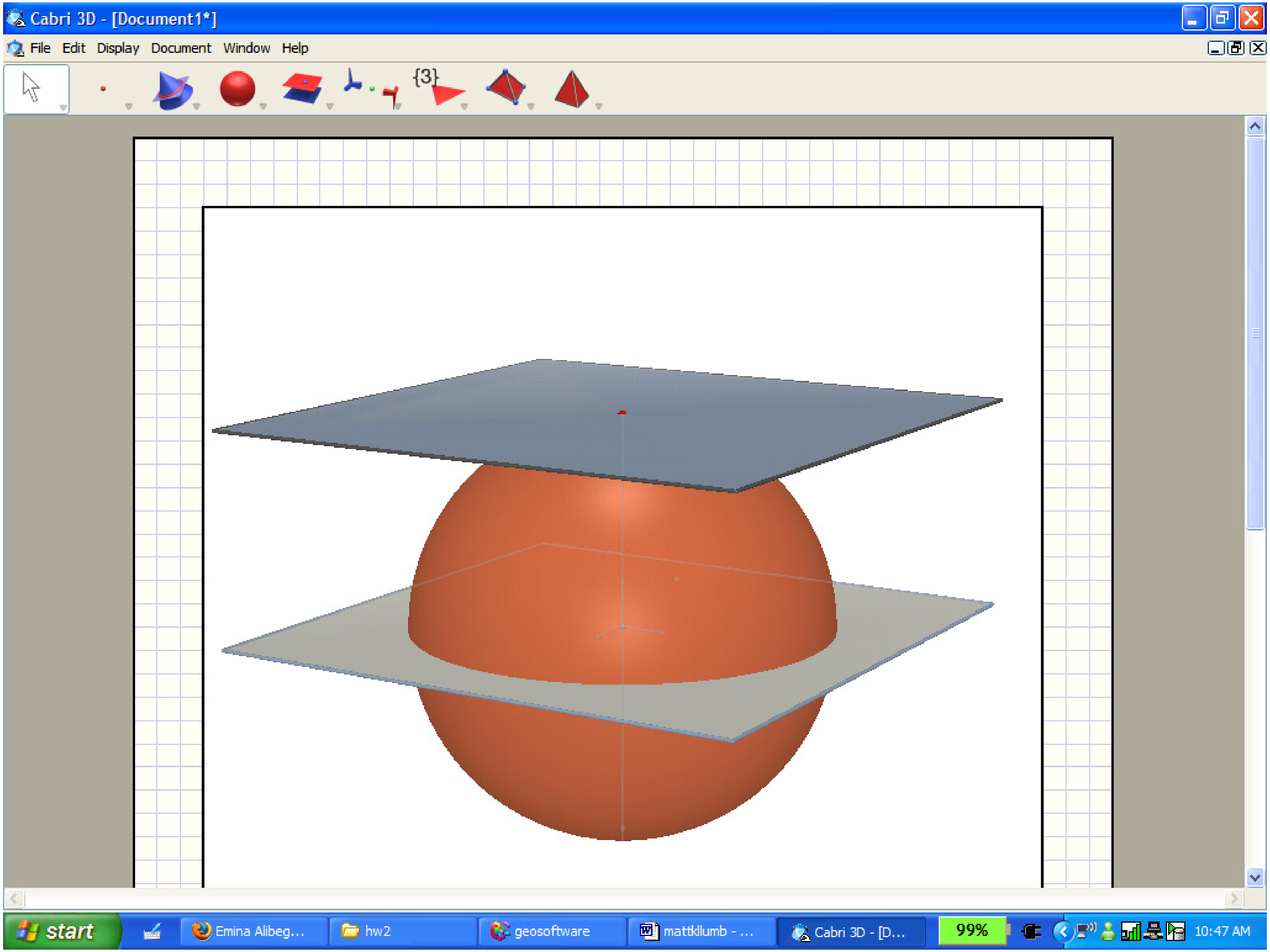
Projective completion of $\mathcal{A}$

- If $\mathcal{A}$ is an affine plane, we enlarge it to $\mathcal{A}^*$ by adding a point $P_{[l]}$ for each equivalence class $[l]$ and we declare that $P_{[l]}$ lies on each line in $[l]$. $P_{[l]}$ is called a point at infinity. We also add a line that consists of all points at infinity and only those points.

Exercise

- What is the projective completion of Cartesian plane?

- It is real projective plane, P^2. To find the text that will go well with following few slides refer to book, page 61, Example 7.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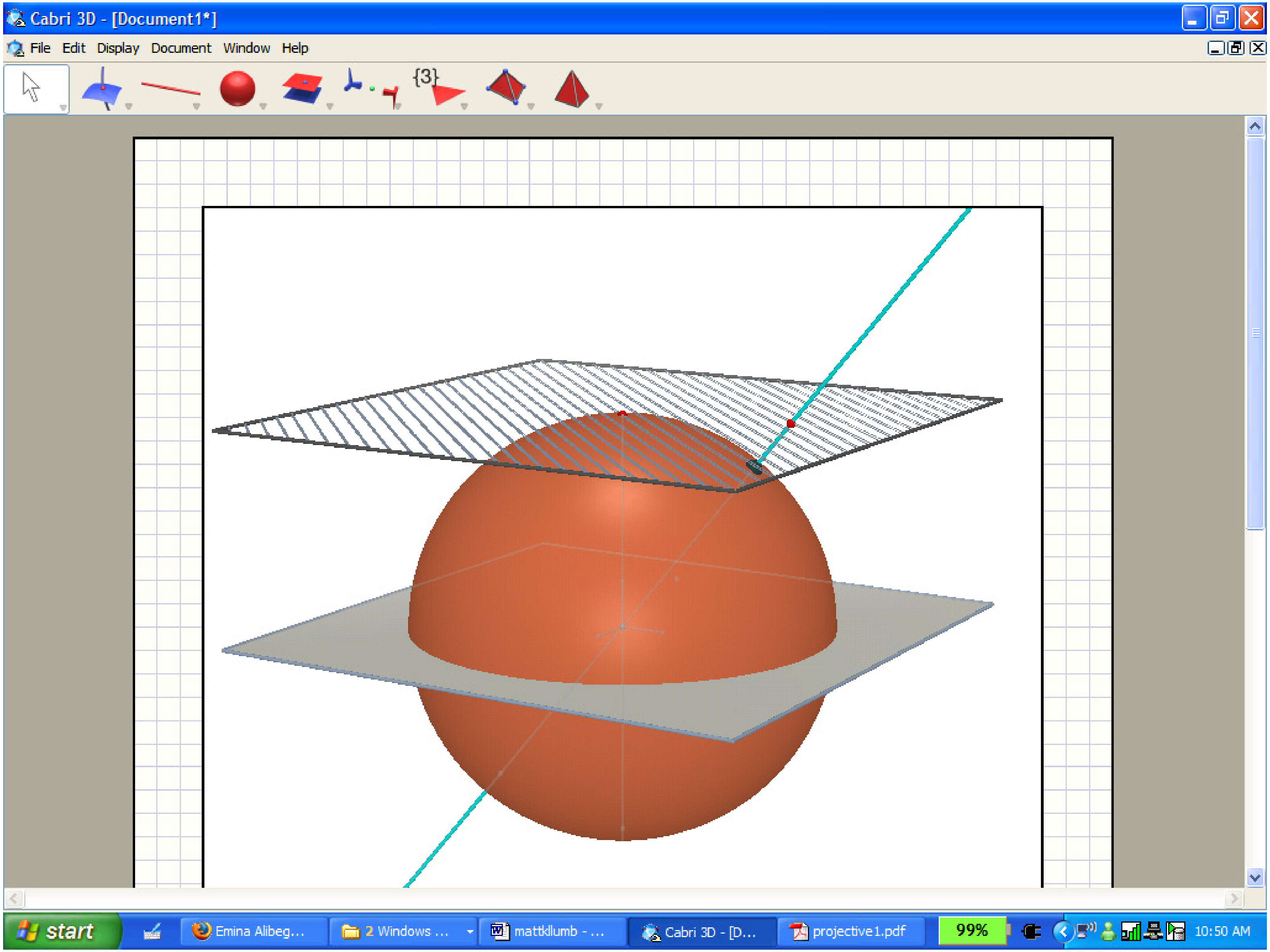
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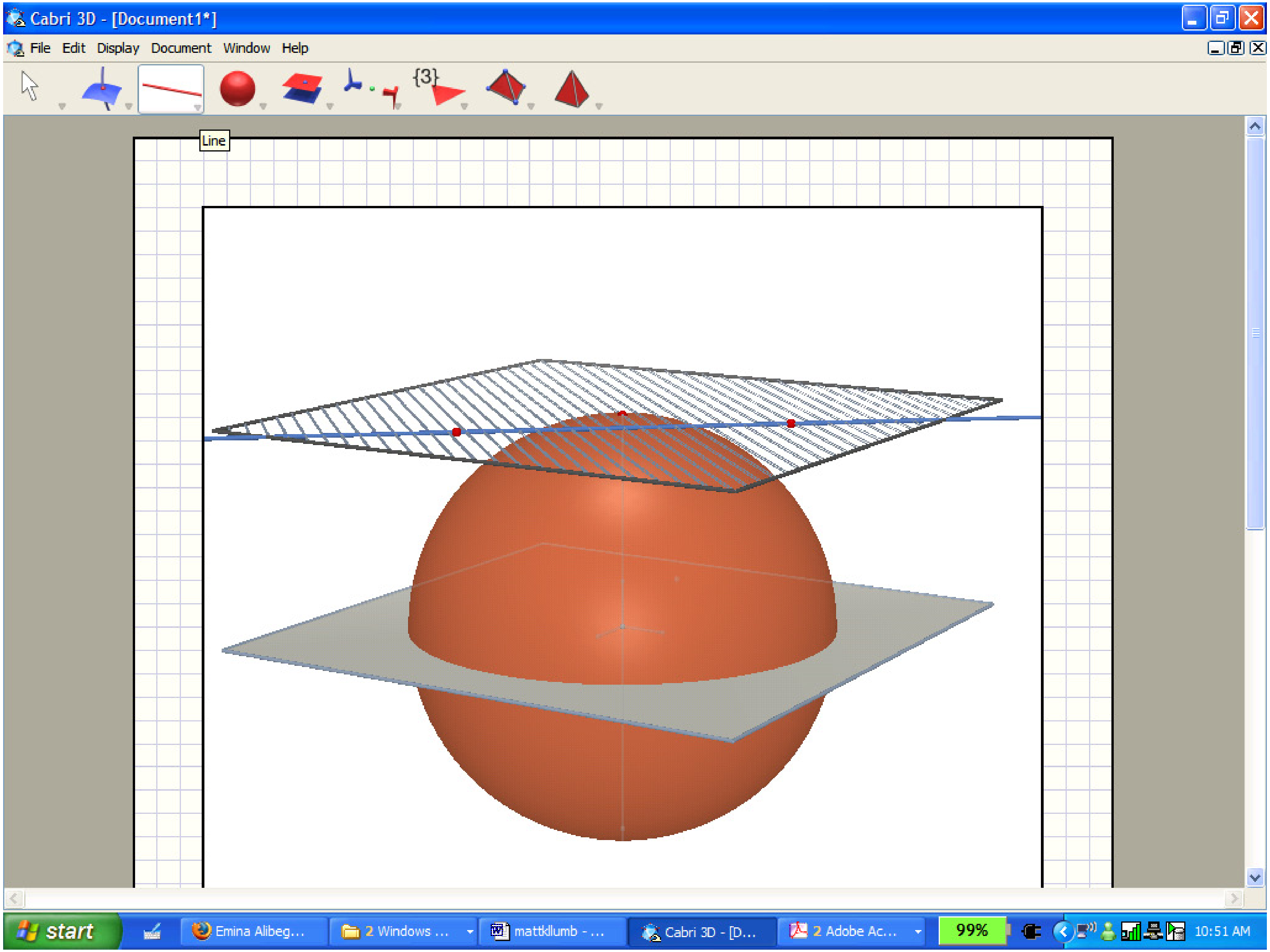
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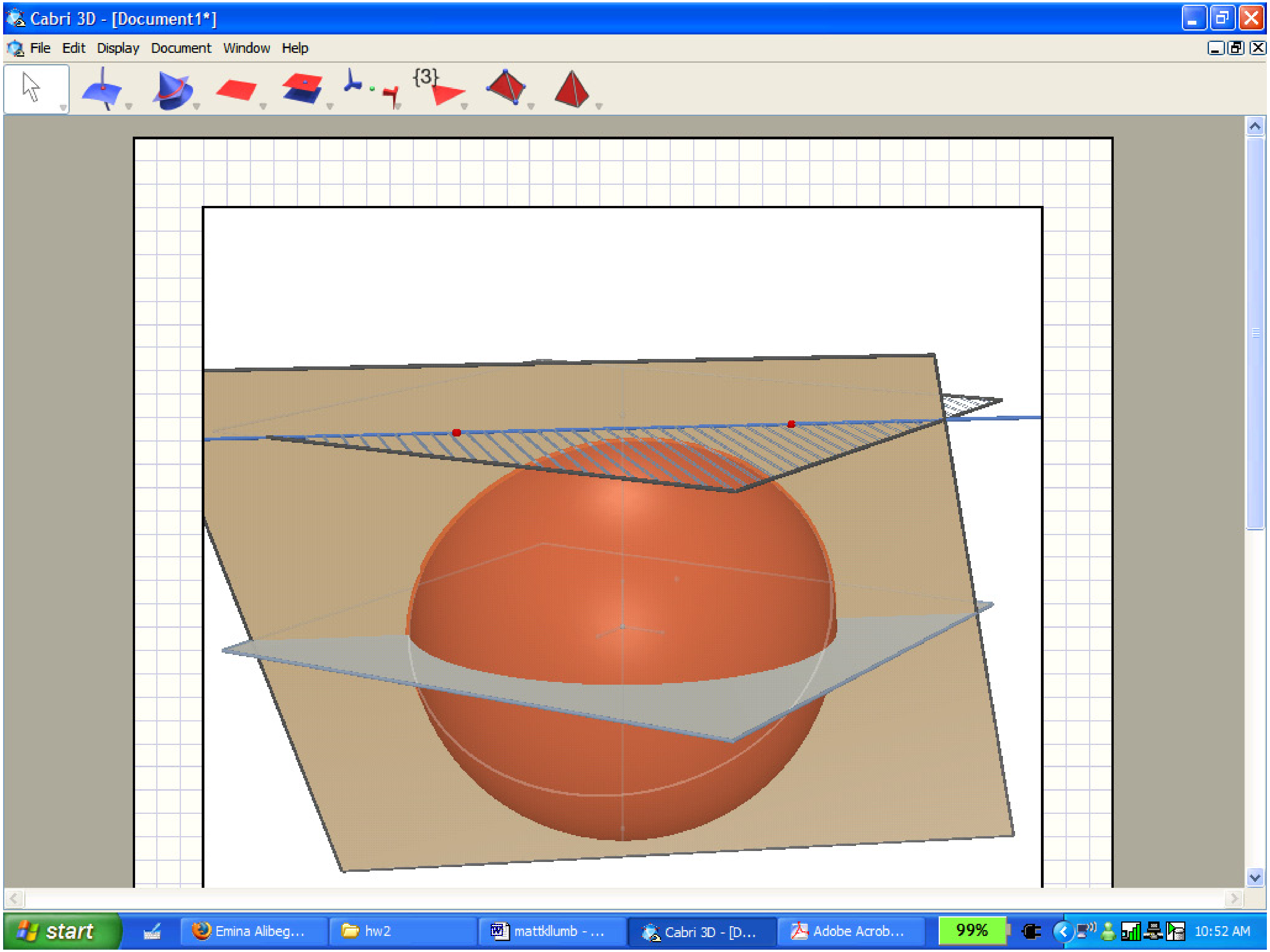
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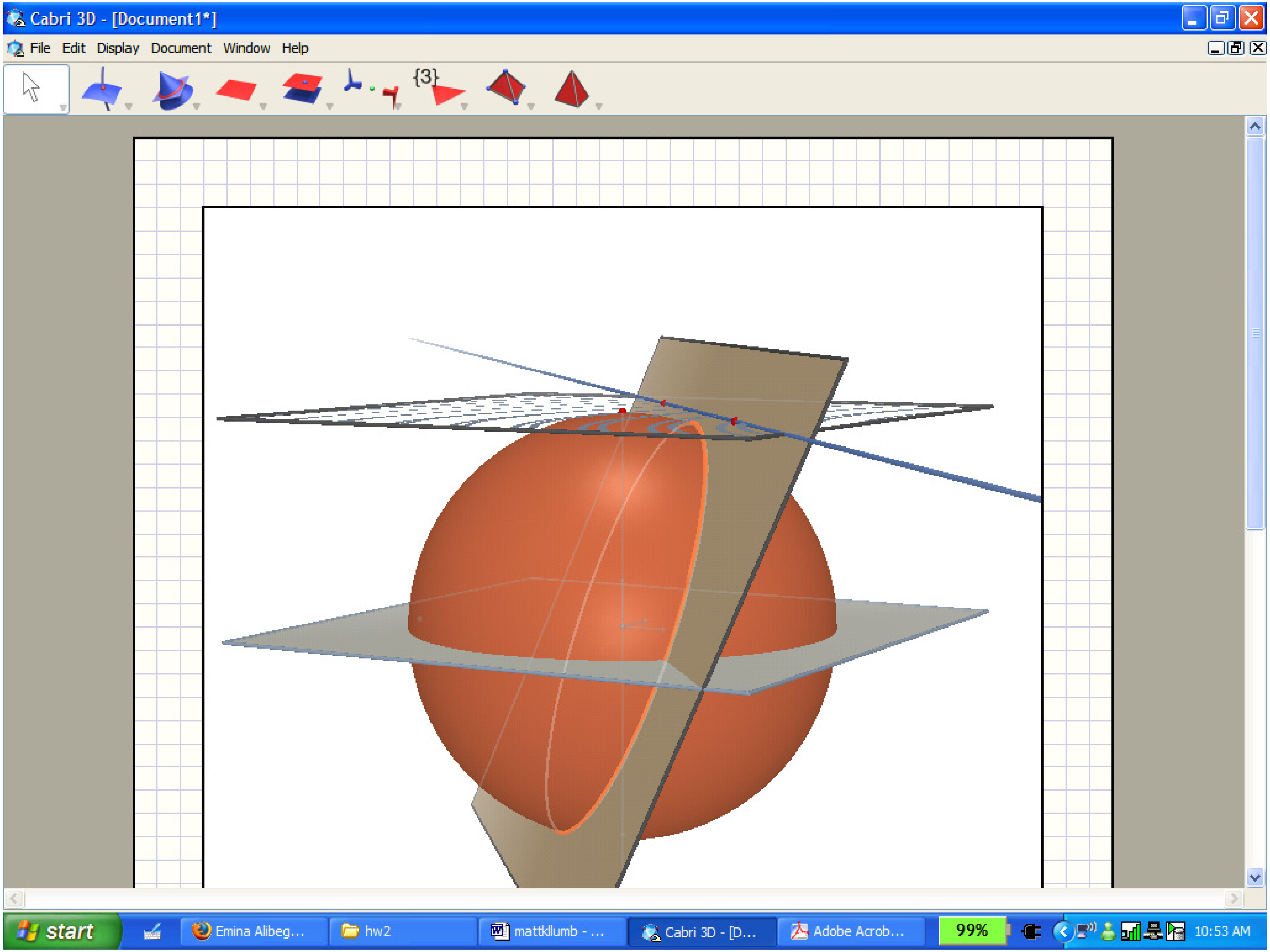
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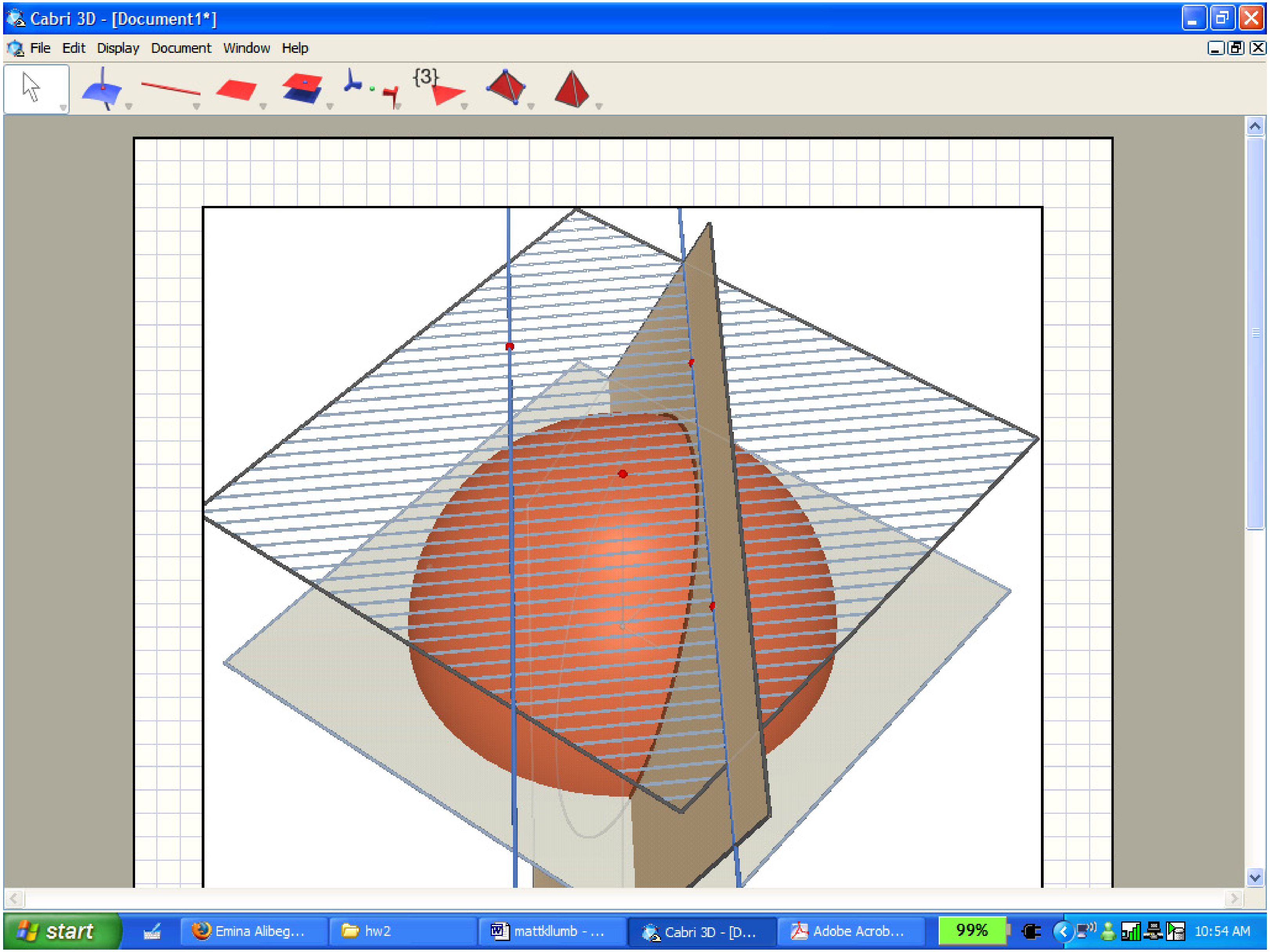

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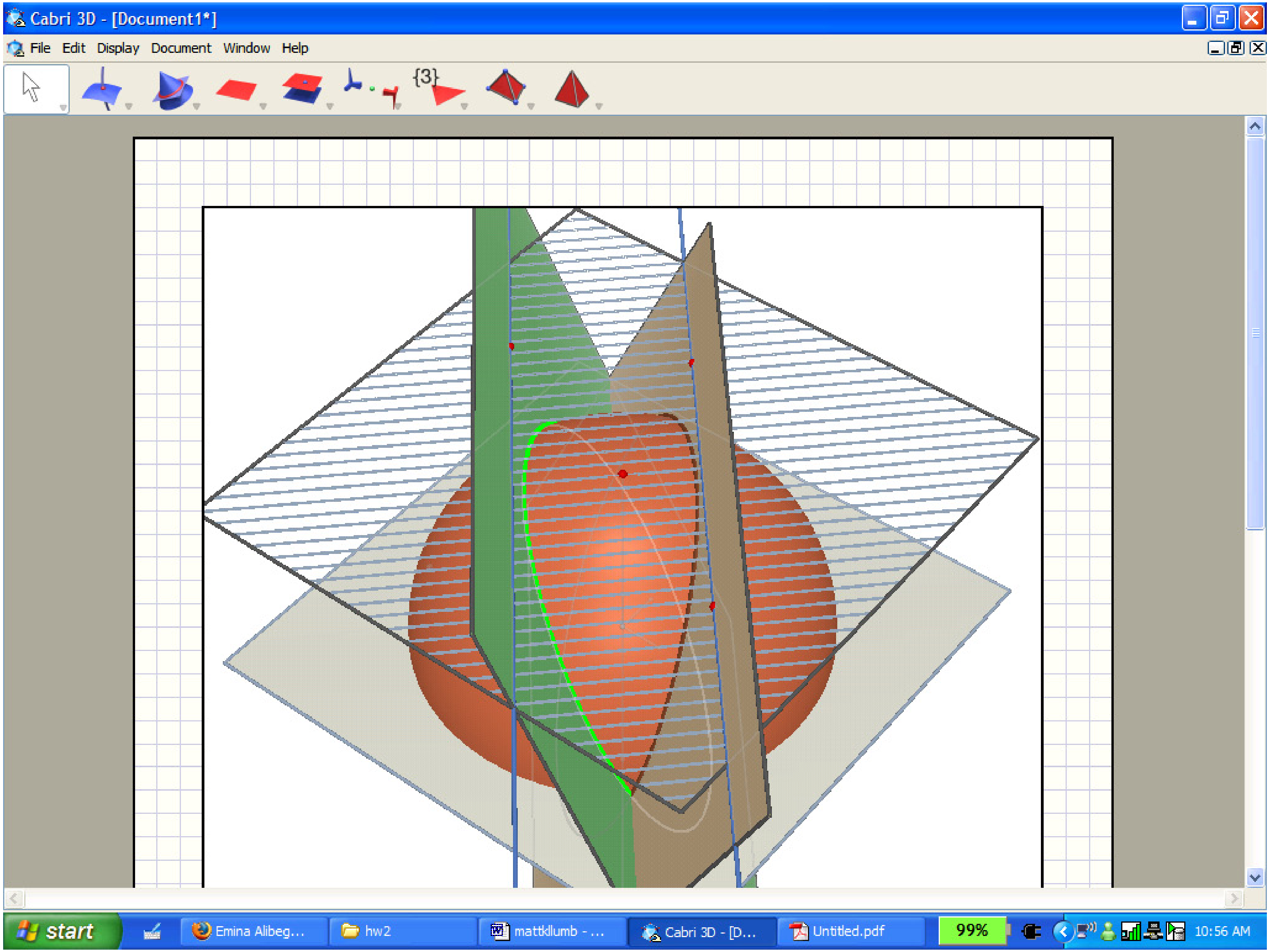
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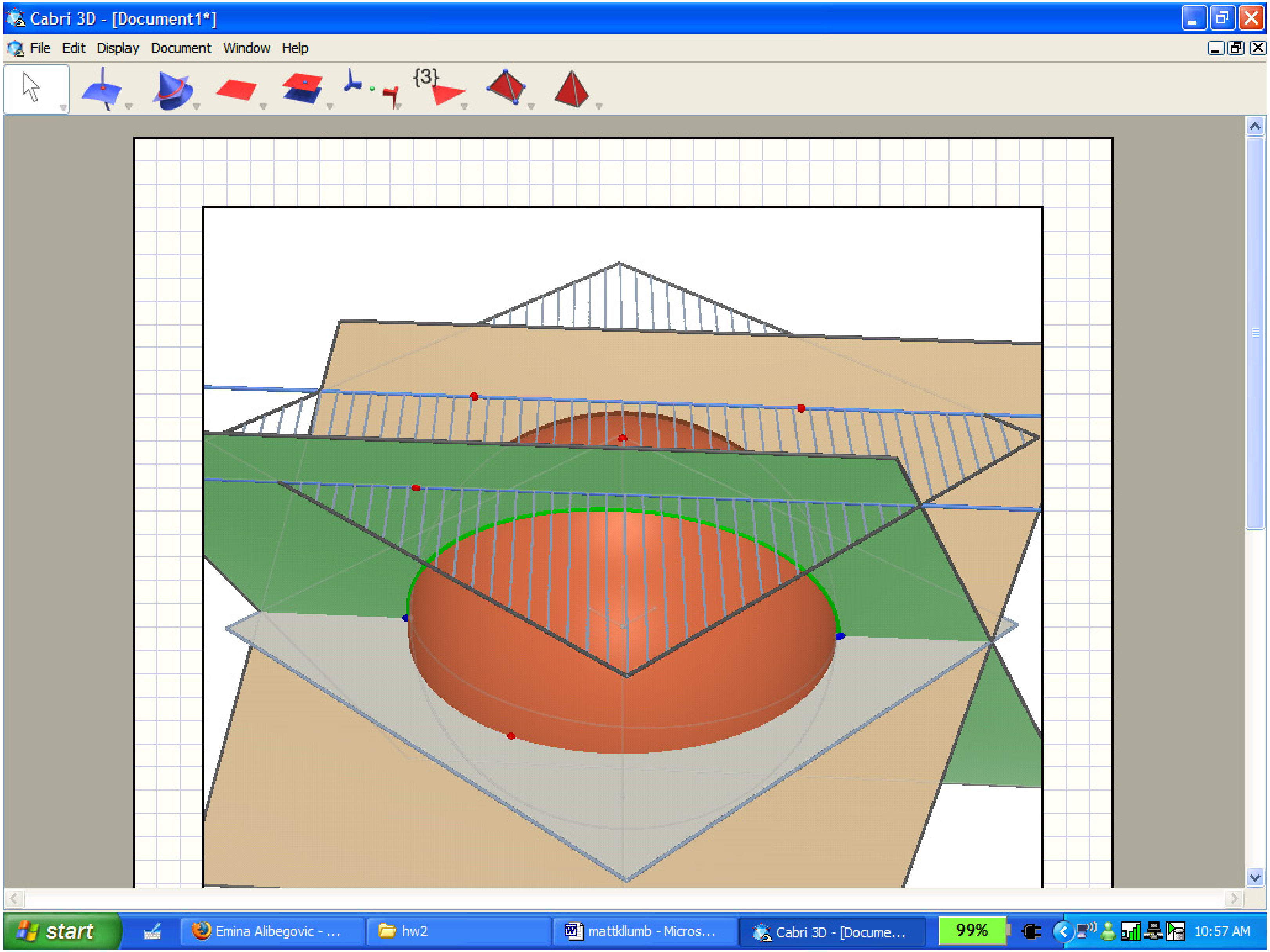

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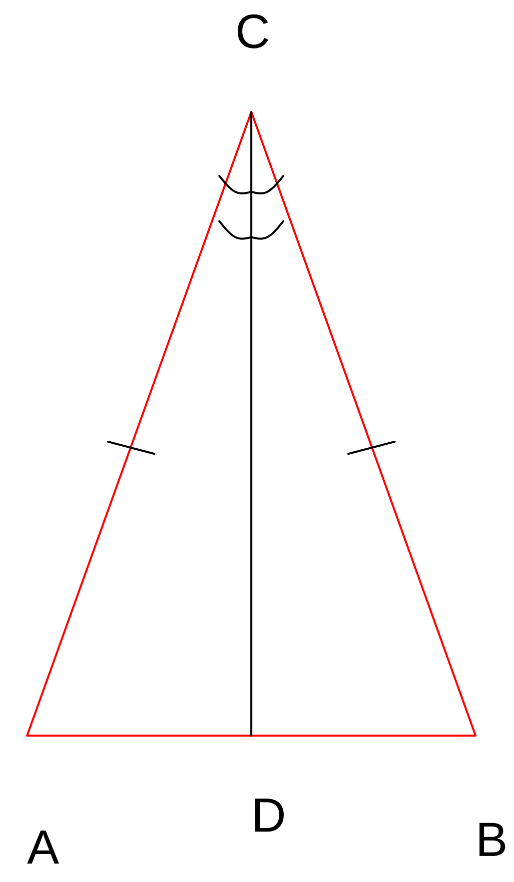
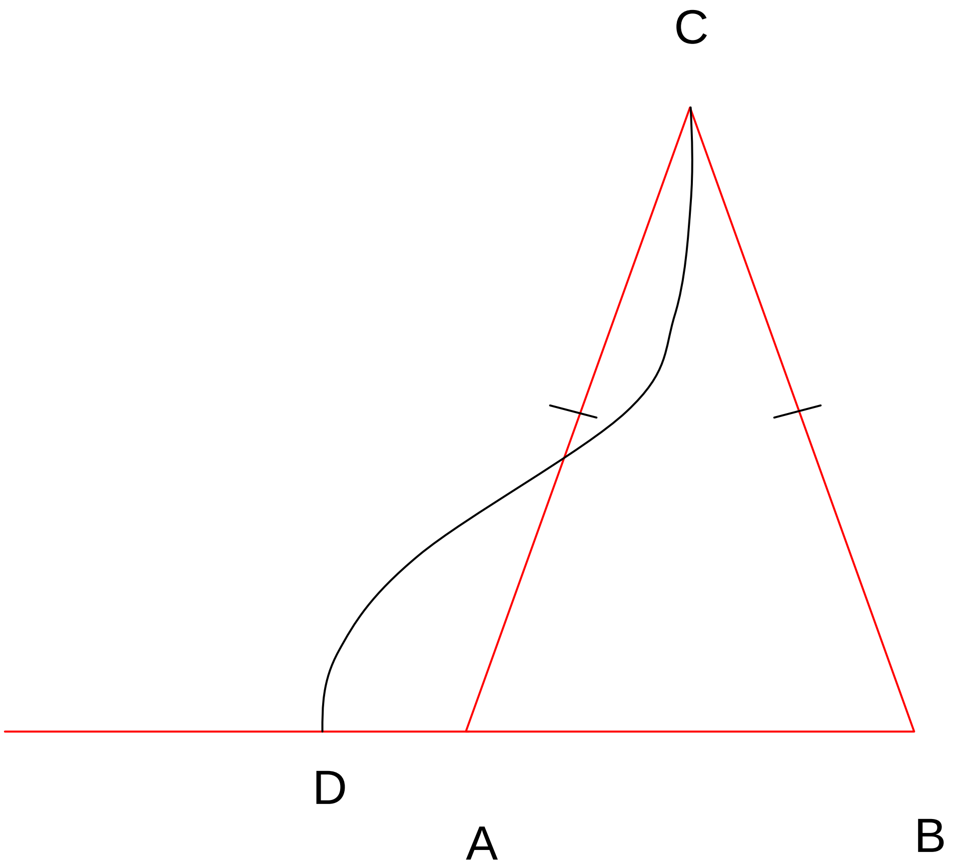
Base angles of isosceles triangle are congruent

Let ABC be a triangle with AC ≅ BC. By Theorem X, ∢C has a bisector. Let the bisector of ∢C meet AB at D. In triangles ACD and BCD, AC ≅ BC by hypothesis. ∢ACD ≅ ∢BCD, by definition of a bisector. Therefore, triangles ACD and BCD are congruent by SAS. Hence, ∢A ≅ ∢B.

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D
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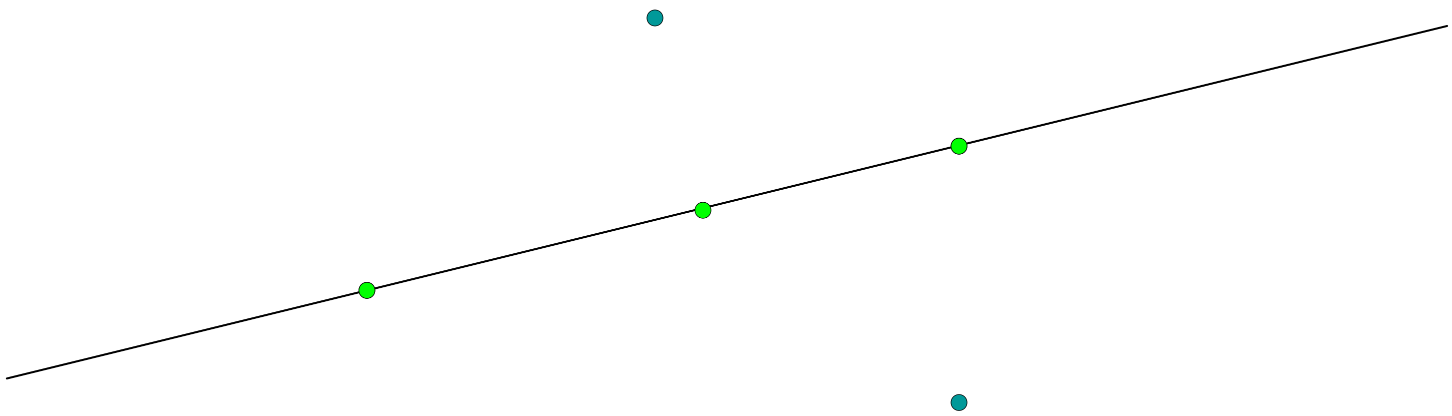
B

Notation

- If P and Q are two distinct points, $\overleftrightarrow{PQ}$ denotes the unique line through P and Q.
- If *l* is the line, {*l* } denotes the set of all points on *l*.
- A*B*C will be an abbreviation for "the point B is between point A and point C"

Betweenness axioms

- Would you say that one of these points is between the other two?



- ***B-1***: If A*B*C, then A, B and C are three distinct points lying on the same line and C*B*A.